

## Rationale establishing why the Indiana State Board of Education should adopt Math in Focus (Singapore math) – Grades K-5

Math in Focus is the US Edition of Singapore's most widely used elementary math program. The Singapore math framework was one of the 15 national curriculums examined by the Common Core State Standards (CCSS) committee and had a particularly important impact on the Common Core writers and contributors due in part to the consistent success Singapore has experienced on international comparison studies over the past 15 years. Both the CCSS and the Singapore mathematics syllabus emphasize a shift towards greater focus and coherence and deep math understanding.

The parallel assumptions behind Singapore math and the CCSS make Math in Focus an ideal choice for Indiana schools. As seen in a document released by Achieve, an independent, nonprofit education reform organization that partnered with NGA and CCSSO on the Common Core State Standards Initiative, the Singapore math syllabus is well aligned to the CCSS:

*"Overall, the CCSS are well aligned to Singapore's Mathematics Syllabus. Policymakers can be assured that in adopting the CCSS, they will be setting learning expectations for students that are similar to those set by Singapore in terms of rigor, coherence and focus."*

- Achieve

Enclosed in this packet is information that details this alignment, with specific examples from the Math in Focus curriculum. Responses to the Standards for Mathematical Content reviews were also provided for standards that received an overall rating less than "3". These were sent in separate packets as part of response "option 1".

### Packet Contents:

- Math in Focus Common Core Alignment Guide, highlighting the overarching parallel assumptions behind Math in Focus and the CCSS
- Math in Focus Alignment to the CCSS Standards for Mathematical Practice
- Math in Focus correlations to the Common Core State Standards (these can also be found in the samples that were previously provided)
- Achieve's whitepaper - *Comparing the Common Core State Standards and Singapore's Mathematics Syllabus*
- Additional information about Math in Focus and Singapore math (blue tab)

Additionally, we have provided a research summary from a recent analysis on Math in Focus student test scores in New Jersey (see blue "Additional Resources" tab). This analysis shows that Math in Focus students made three times the average gains as the district comparison group, highlighting that a Singapore math curriculum is also effective in US classrooms.

We truly hope that you will consider Math in Focus as a choice for Indiana schools. If you have any questions, please contact Laura Rockefeller at 512-721-7204.

# ALIGNMENT GUIDE



## Singapore Math: *Math in Focus*<sup>™</sup> and the Common Core State Standards Alignment Guide

The Common Core State Standards for Mathematics is an initiative towards more focused grade level standards.

The research base used to guide the Common Core State Standards noted conclusions from TIMSS, where Singapore has been a top scoring nation for 15 years. Apparent in the TIMSS and other studies of high-performing countries is a more coherent and focused curriculum. The Singapore math framework was one of the 15 national curriculums examined by the committee and had a particularly important impact on the Common Core writers and contributors.

What follows is a guide that identifies the parallel assumptions behind *Math in Focus*—the U.S. Edition of Singapore's most widely-used program—and the Common Core State Standards.

### ► Curriculum must be focused and coherent

#### Common Core State Standards:

For over a decade, research studies of mathematics education in high performing countries have pointed to the conclusion that the mathematics curriculum in the United States must become substantially more focused and coherent in order to improve mathematics achievement in this country.

*(Common Core State Standards for Mathematics, 3)*

- *Math in Focus* is organized to teach fewer topics in each grade, but to teach them thoroughly. When a concept appears in a subsequent grade level, it is always at a higher level. For instance, *Math in Focus* teaches fractions in a way that builds from grade to grade, where mastery of grade-appropriate concepts eliminates the need for repetition year after year. Fractions are not taught in first grade, to allow students the time they need to master the whole number concepts that form the basis of fractions. Fractions are introduced in second grade, which covers what a fraction is. In third grade, students cover equivalent fractions and fractions of a set. Fourth grade deals with mixed fractions and addition of simple fractions. Finally, fifth grade moves on to addition, subtraction, and multiplication of fractions as well as division of fractions by whole numbers. Each grade level addresses an increasingly complex facet of fractions, and draws on the mastery of concepts that has been developed in previous years. This is the coherence and focus that the Common Core State Standards call for.

**Math in  
Focus**<sup>™</sup>

The Singapore Approach

**GREAT  
SOURCE.**

HOUGHTON MIFFLIN HARCOURT



▶ **Focus on number, geometry, and measurement in elementary grades**

**Common Core State Standards:**

Mathematics experiences in early childhood settings should concentrate on (1) number (which includes whole number, operations, and relations) and (2) geometry, spatial relations, and measurement, with more mathematics learning time devoted to number than to other topics.

*(Common Core State Standards for Mathematics, 3)*

- ▶ ***Math in Focus*** emphasizes number and operations in every grade, K-5, just as recommended in the Common Core State Standards. The textbook is divided into two books, roughly a semester each. Approximately 75% of Book A is devoted to number and operations and 60-70% of Book B to geometry and measurement where the number concepts are practiced, connected, and applied. The key number topics are in the beginning of the school year so students have a whole year to master them.



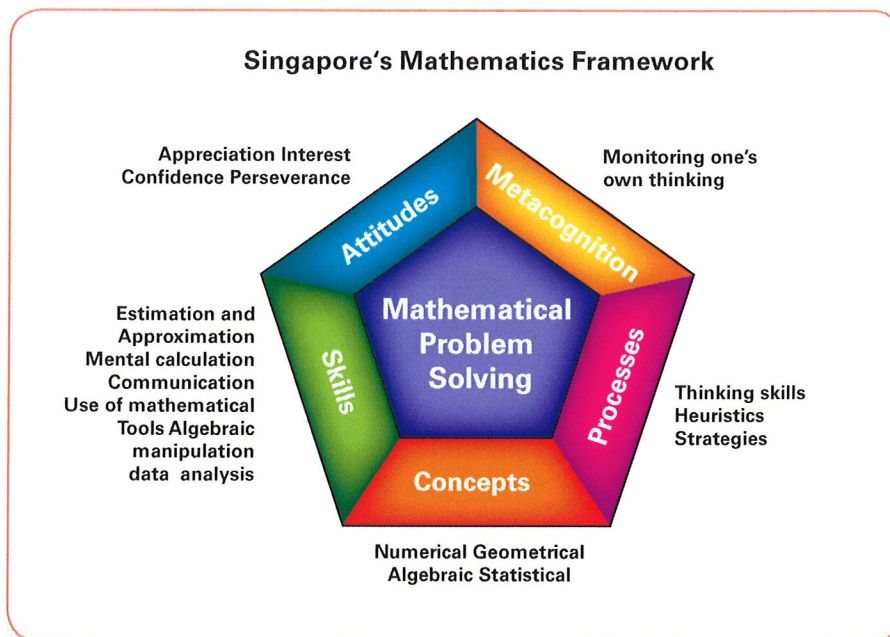
► **Curriculum must include both conceptual understanding and procedural fluency**

**Common Core State Standards:**

The Standards for Mathematical Content are a balanced combination of procedure and understanding.

*(Common Core State Standards for Mathematics, 8)*

- **Math in Focus** is built around the Singapore Ministry of Education's famous pentagon that emphasizes conceptual understanding, skill development, strategies for solving problems, attitudes towards math, and metacognition that enables students to become excellent problem solvers. The highly visual nature of the text and the consistent concrete to visual to abstract approach enables all students to both understand how procedures work and to fluently apply them to solve problems.



- **Math in Focus** follows the Singapore Mathematics Framework pentagon. Students are encouraged to consider how they think, how they communicate, and how they solve problems so they can apply their skills to subsequent problems.

# **MATHEMATICAL PRACTICE**

## **Alignment to the Standards for Mathematical Practice**

Math in Focus aligns to the Common Core State Standards Standards for Mathematical Practice throughout the grade levels. The following document highlights this alignment and provides an example for each grade. Additional examples can also be found by referring to the Focus on the Common Core Teacher's Edition tabs that were provided with the Math in Focus vault samples.

### **Works Cited:**

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# 1) Make sense of problems and persevere in solving them.

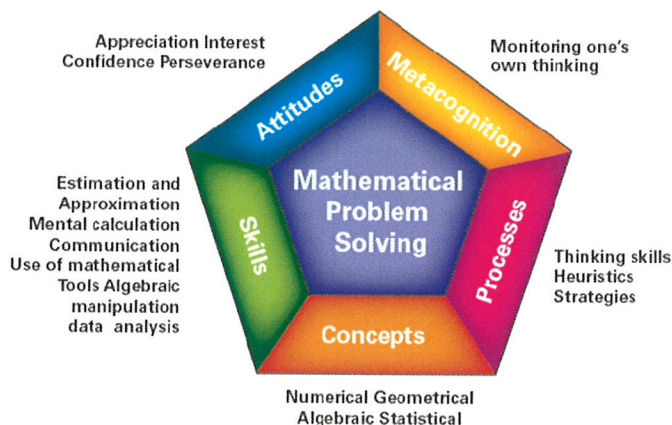
## From the Common Core State Standards:

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

## How Math in Focus Aligns:

Math in Focus is built around the Singapore Ministry of Education’s famous pentagon, which places mathematical problem solving at the core of the curriculum. Around it are the skills and knowledge needed to develop successful problem solvers, with concepts, skills and processes building a foundation for attitudes and metacognition. Math in Focus is based on the premise that in order for students to persevere and solve both routine and non-routine problems, they need to be given tools that they can use consistently and successfully. They need to understand both the how and the why of math so that they can self-monitor and become empowered problem solvers. This in turn spurs positive attitudes that allow students to solidify their learning and enjoy mathematics. Math in Focus teaches content through a problem solving perspective. Strong emphasis is placed on the concrete-pictorial-abstract progress to solve and master problems. This leads to strong conceptual understanding.

Singapore’s Mathematics Framework



## Examples throughout the Math in Focus curriculum:

Throughout the Math in Focus program, you will find problem solving at the heart of the curriculum. In addition to solving problems in the Learn, Guided Practice, Let's Practice and independent practice portions of each lesson, Put on your Thinking Cap! Problems (grades 1-5) and the Grade K Student Books challenge students to put the skills they've learned to work finding solutions in non-routine situations.

**Count and write.**

How many more socks do we need?

Three!

\_\_\_\_\_ more flowers are needed.

\_\_\_\_\_ more flowers are needed.

\_\_\_\_\_ more flowers are needed.

Kindergarten:

Student Book A, Part 2, page 22

Younger students use concrete and pictorial models to solve problems like the ones pictured here.

**CRITICAL THINKING SKILLS**  
**Put On Your Thinking Cap!**

**PROBLEM SOLVING**  
**Find the number of beads.**  
**Use number bonds to help you.**

1 There are 6 beads under the two cups.

2 There are 8 beads under the two cups.

3 There are 10 beads under the three cups.

**ON YOUR OWN**  
Go to Workbook A:  
Put on Your Thinking Cap!  
pages 31-32

1<sup>st</sup> Grade:

Student Book A, page 37

1st grade students use number bonds to determine unknowns, promoting early algebraic thinking through relevant problem solving.

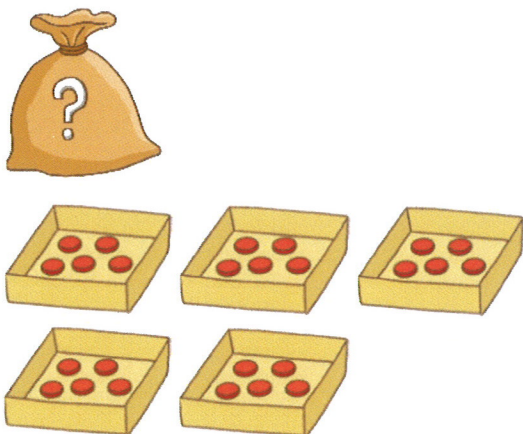


**CRITICAL THINKING SKILLS**  
**Put On Your Thinking Cap!**

**PROBLEM SOLVING**

**Solve.**

Meena has 28 counters.  
She puts some in a bag.  
She puts the rest of the counters into 5 boxes.  
If each box contains 5 counters, how many counters are in the bag?



There are  counters in the bag.

**ON YOUR OWN**

Go to Workbook B:  
Put On Your Thinking Cap!  
pages 173-174

2<sup>nd</sup> Grade:

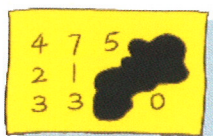
Student Book B, page 217

In order to solve problems like the one pictured here, students cannot simply memorize. Rather, they need to understand how the math works and be able to manipulate it to solve non-routine problems.

**CRITICAL THINKING SKILLS**  
**Put On Your Thinking Cap!**

**PROBLEM SOLVING**

Rita wrote three 4-digit numbers on a sheet of paper.  
She accidentally spilled some ink on the paper.  
Some digits were covered by the ink.  
Using the clues given, help Rita find the digits covered by the ink.



**CLUES**

The sum of all the ones is 17.

The ones digit of the first number is the greatest 1-digit number.

The digit in the tens place of the second number is one more than the digit in the tens place of the first number.

The tens digit of the third number is 4 less than the tens digit of the second number.

**ON YOUR OWN**

Go to Workbook A:  
Put On Your Thinking Cap!  
pages 17-18

3<sup>rd</sup> Grade:

Student Book A, page 32

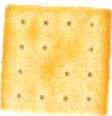
Non-routine problems like this one emphasize the necessity of understanding how the math works. Math in Focus teaches students to explore the meaning of operations so they can go beyond simply identifying a symbol to determine which operation to use. Instead, students are challenged to think about the situation and choosing the operation based on reason and its application to the problem.





**CRITICAL THINKING SKILLS**  
**Put On Your Thinking Cap!**

**PROBLEM SOLVING**

2 Jessie had a whole graham cracker.  
Minah had only part of another graham cracker.  
Jessie gave  $\frac{1}{2}$  of her graham cracker to Minah.  
In the end, both girls had the same fractional part of a graham cracker.  
What fraction of a graham cracker did Minah have at first?




Jessie: 

Minah: 

Here are 2 equal bars to show that both of them had an equal portion of a graham cracker in the end.

Work backward to find the fraction of the graham cracker Minah had at first.



**ON YOUR OWN**  
Go to Workbook A:  
Put on Your Thinking Cap!  
pages 169-170

Chapter 6 Fractions and Mixed Numbers 269

4<sup>th</sup> Grade:

Student Book A, page 269

As students get older, problems become more and more complex, but consistent problem solving tools such as bar modeling give students the tools they need to persevere in solving them.

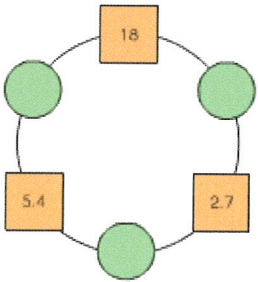
Thought bubbles also help students monitor their work and assess whether or not they are on the right track and their answers make sense.

**CRITICAL THINKING SKILLS**  
**Put On Your Thinking Cap!**

**PROBLEM SOLVING**

**Solve these problems.**

1 The number in the square is the product of the numbers in the two circles next to it. Find the numbers in the circles.



2 Simone bought a total of 10 birthday hats and noisemakers. Each birthday hat cost \$1.50 and each noisemaker cost \$2.50. The noisemakers cost \$13 more than the birthday hats. How many of each item did she buy?

**ON YOUR OWN**  
Go to Workbook B:  
Put on Your Thinking Cap!  
pages 51-54

Chapter 9 Multiplying and Dividing Decimals 31

5<sup>th</sup> Grade:

Student Book B, page 81

By the time students reach 5th grade, they have developed the confidence and skills needed to become successful problem solvers. Because they have consistently been exposed to non-routine problems, they are ready to tackle on to the challenges of middle school and enjoy mathematics.



## 2) Reason abstractly and quantitatively.

### From the Common Core State Standards:

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

### How Math in Focus Aligns:

Math in Focus' concrete to pictorial to abstract progression helps students effectively contextualize and decontextualize situations by developing a deep mastery of concepts. Each topic is approached with the expectation that students will understand both how it works, but also why. Students start by experiencing the concept through hands-on manipulative use. Then, they must translate what they learned in the concrete stage into a visual representation of the concept. Finally, once they have built a strong understanding, they are able to represent the concept abstractly. Once students reach the abstract stage, they have had enough exposure to the concept and they are able to manipulate it and apply it in multiple contexts. They are also able to extend and make inferences; this prepares them for success in more advanced levels of mathematics. They are able to both use the symbols but also understand why they work, which allows students to relate them to other situations and apply them effectively.

### Examples throughout the Math in Focus curriculum:

#### Let's Talk!

1. Give children a set of connecting cubes. Ask children to make one short line of cubes, and one long line of cubes.

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2. Give children a set of connecting cubes. Ask children to make three lines of cubes of different lengths. Then, ask them to point to the longest line, and then the shortest line.

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3. Give children a set of attribute blocks and ask them to sort the blocks first by color, and then by shape.

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4. Show children 8 connecting cubes.  
Say: There are 8 cubes.  
Take away 3 cubes  
Ask: How many are left? (5)  
Repeat by varying the number of cubes present and cubes taken away.

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5. Ask two children for their pencil cases or use other classroom containers.  
Ask: Which do you think holds more? Which do you think holds less?  
Empty their pencil cases and fill them with connecting cubes to check their answers.


Kindergarten:

Assessments Book, page 35

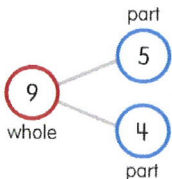
In "Let's Talk!" portions of the Kindergarten program, students make sense of the math they are learning through hands-on activities and meaningful discussion.



How many beanbags are on the floor?



$9 - 5 = ?$



$9 - 5 = 4$

There are 4 beanbags on the floor.

1<sup>st</sup> Grade:

Student Book A, page 74

1st grade students model subtraction concretely by starting with physical beanbags. They then move on to the pictorial stage, using number bonds to represent the action of taking away. Finally, they write subtraction symbolically. This concrete to pictorial to abstract progression helps students truly make sense of the quantities.

#### PROBLEM SOLVING

Find the missing numbers in each box.

1

-	1
<hr/>	
8	8

2

-	4	4
<hr/>		
4	4	4

3

6	5	4
-	2	
<hr/>		
4	2	0

2<sup>nd</sup> Grade:

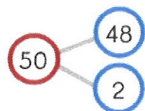
Student Book A, page 88

After a lesson on subtraction up to 1,000, 2nd grade students need to have a deep enough understanding in order to recognize situations where the “-” sign doesn’t necessarily require taking away to solve. Even though these look like simple subtraction problems, students

need to understand how each number is functioning in order to fill in the green squares.

**Add 2-digit numbers mentally using the ‘add the tens, then subtract the extra ones’ strategy.**

Find  $34 + 48$ .



**Step 1** Add 50 to 34.

$$34 + 50 = 84$$

**Step 2** Subtract 2 from the result.

$$84 - 2 = 82$$

So,  $34 + 48 = 82$ .

Do you know why you add 50 and then subtract 2?



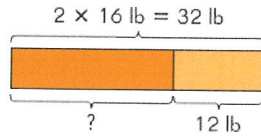
3<sup>rd</sup> Grade:

Student Book A, page 42

Math in Focus teaches students ways to break apart numbers to compute mentally. This requires students to develop an understanding of the meaning of quantities. In this example, students learn that to add  $34 + 48$  is the same as adding  $50 + 34$  and then subtracting 2. A thought bubble reinforces this reasoning ability by asking students *why* they would manipulate the numbers like this.

# Learn Solve problems using the mean.

The mean weight of 2 tables is 16 pounds. The weight of one of the tables is 12 pounds. What is the weight of the other table?



$$\begin{aligned} \text{Total weight of the 2 tables} &= 16 \times 2 \\ &= 32 \text{ lb} \end{aligned}$$

$$\begin{aligned} \text{Weight of the other table} &= 32 - 12 \\ &= 20 \text{ lb} \end{aligned}$$

The weight of the other table is 20 pounds.

4<sup>th</sup> Grade:

Student Book A, page 204

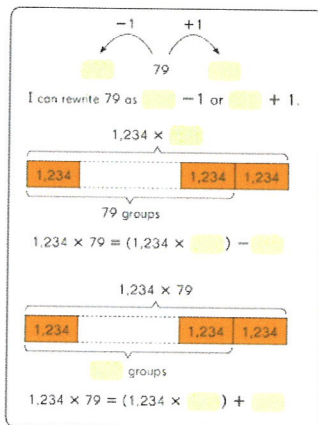
To solve problems like this one, students must be able to take their understanding of mean and consider how it is used to find weight. This goes beyond simply computing mean to using abstract and quantitative reasoning to consider how it relates to other operations.

## CRITICAL THINKING SKILLS Put On Your Thinking Cap!

### PROBLEM SOLVING

The **9** key on the calculator is not working.

Explain how you can still use the calculator to find  $1,234 \times 79$  in two ways.



#### ON YOUR OWN

Go to Workbook A:  
Put on Your Thinking Cap,  
pages 75–78

5<sup>th</sup> Grade:

Student Book A, page 109

Put on your Thinking Cap! problems like this one challenge students to consider what quantities mean, how they are composed and how they can use model drawing to represent a solution. 5th grade students are taught to think flexibly about numbers so that they can be deconstructed if needed to solve a problem. Here, while students may understand what 9 means, they must also understand how it can be manipulated in order to solve. Math in Focus provides the tools and guidance students need to develop this abstract thinking.



### 3) Construct viable arguments and critique the reasoning of others.

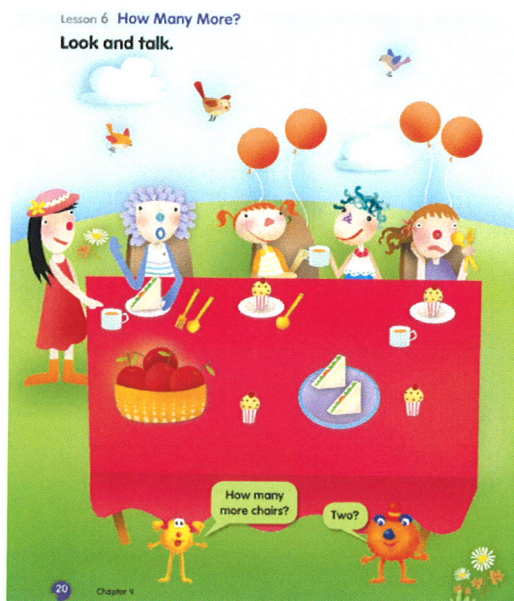
#### From the Common Core State Standards:

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

#### How Math in Focus Aligns:

As seen on the Singapore Mathematics Framework pentagon, metacognition is a foundational part of the Singapore curriculum. Students are taught to self-monitor, so they can determine whether or not their solutions make sense. Journal questions and other explicit opportunities to explain their thinking are found throughout the program and students are systematically taught to use visual diagrams to represent mathematical relationships in such a way as to not only accurately solve problems, but also to justify their answers. Chapters conclude with a Put on Your Thinking Cap! problem; this is a comprehensive opportunity for students to apply concepts and present viable arguments. Games, explorations and hands-on activities are also strategically placed in chapters when students are learning concepts. During these collaborative experiences, students interact with one another to construct viable arguments and critique the reasoning of others in a constructive manner. In addition, “thought bubbles” guide students like a tutor throughout the entire student edition. These scaffolded dialogues help students articulate concepts, check for understanding, analyze, justify conclusions, and self regulate if necessary.

#### Examples throughout the Math in Focus curriculum:



Kindergarten:

Student Book A, Part 2, page 20

Kindergarten lessons begin with group discussions where teachers prompt students to discuss the math that can be seen in a Big Book or Student Book scene. This provides an ideal opportunity for students to verbally explain their thinking and construct age-appropriate arguments.



**READING AND WRITING MATH**  
**Math Journal**

Tania completes this number pattern.  
32, 33, 34, 35, 36, 37, 38, 39

She explains how she found each number in the pattern.

I added 1 to 32 to get 33.  
I added 1 to 33 to get 34.  
I just have to add 1 to get the next number.

33 is 1 more than 32.  
34 is 1 more than 33.

$32 + 1 = 33$   
 $33 + 1 = 34$

How do you find the missing numbers in this pattern?  
40, 30, , 10,

In the pattern, is the next number more or less?

1<sup>st</sup> Grade:

Student Book B, page 74

1st grade students learn how to explain their reasoning through guided math journal exercises and modeled thought processes.

**Check!**

$3 + 5 = 8$   
 + 8 =   
Is the answer correct?

2<sup>nd</sup> Grade:

Student Book A, page 104

Throughout the program, students are taught to check their answers and make sure their solutions are reasonable. Look for the “Check!” icon throughout the Student Books.

**READING AND WRITING MATH**  
**Math Journal**

List the steps to arrange the numbers in order from least to greatest.

Example

1,984 2,084 1,884

- I compare the thousands
- I can see that 2,084 is the greatest
- I compare the hundreds
- I can see that 1,884 is the least number.

Arranged from least to greatest:  
1,884 1,984 2,084  
least

9,049 9,654 8,785

Arranged from least to greatest:

List the steps to get your answer.

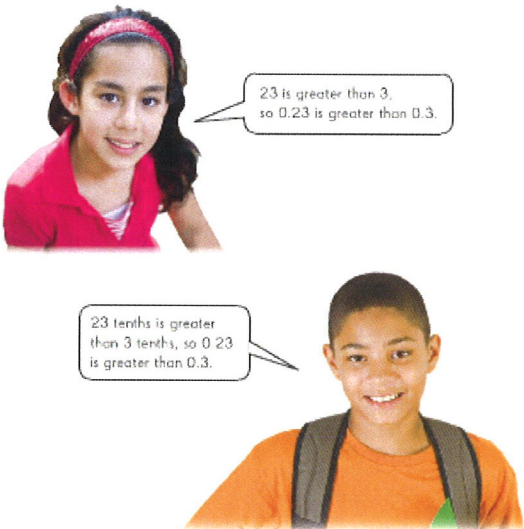
3<sup>rd</sup> Grade:

Student Book A, page 31

Exercises that require students to list the steps they take to get an answer help develop the language students need to explain how they solved a problem and justify their solutions.

**READING AND WRITING MATH**  
**Math Journal**

Both Andy and Rita think that 0.23 is greater than 0.3.



23 is greater than 3, so 0.23 is greater than 0.3.

23 tenths is greater than 3 tenths, so 0.23 is greater than 0.3.

Do you agree? Why or why not? Explain your answer.

4<sup>th</sup> Grade:

Student Book B, page 33

This 4th grade example presents students with a situation and asks them whether or not they agree. This prompts students to construct an argument to support their answer and provides opportunities for classroom discussion.



**Some problems require two steps to solve.**



The Fairfield Elementary School library is in the shape of a rectangle. It measures 36 yards by 21 yards. The school's principal, Mr. Jefferson, wants to carpet the library floor. Find the cost of carpeting the library fully if a 1-square-yard carpet tile costs \$16.

First, find the floor area of the library.

$$\begin{aligned}\text{Area} &= \text{length} \times \text{width} \\ &= 36 \times 21 \\ &= 756 \text{ yd}^2\end{aligned}$$

Estimate the answer.  
36 rounds to 40.  
21 rounds to 20.  
 $40 \times 20 = 800$   
756 is a reasonable answer.

The floor area of the library is 756 square yards.

Then, find the cost of carpeting.

$$\begin{aligned}\text{Cost of carpeting} &= \text{area} \times \text{cost of 1 yd}^2 \\ &= 756 \times \$16 \\ &= \$12,096\end{aligned}$$

Estimate to check if the answer is reasonable.

It will cost \$12,096 to carpet the library fully.

5<sup>th</sup> Grade:

Student Book A, page 98

Throughout Math in Focus, students are asked to estimate to evaluate whether or not their answers are reasonable. This develops the metacognitive skills that are highlighted in the Singapore mathematics pentagon framework and promotes the ultimate goal of developing effective problem solvers.



## 4) Model with mathematics.

### From the Common Core State Standards:

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

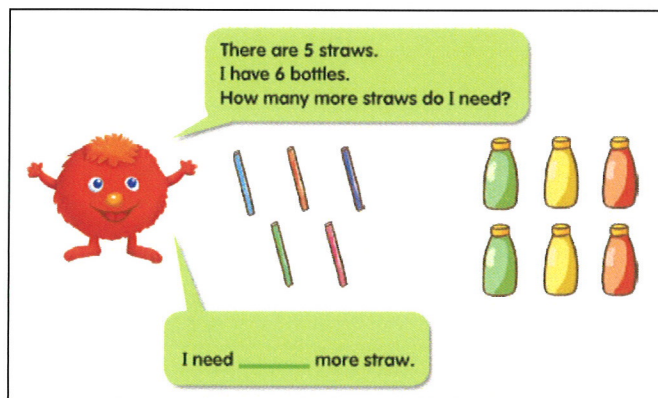
### How Math in Focus Aligns:

Math in Focus follows a concrete to pictorial to abstract progression, introducing concepts first with physical manipulatives or objects, then moving to pictorial representation and finally on to abstract symbols. A number of models are found throughout the program that supports the pictorial state of learning.

Math in Focus places a strong emphasis on number and number relationships, using place value manipulatives and place value charts to model concepts consistently throughout the program. In all the grades, operations are modeled with place value materials so students understand how the standard algorithms work. Even the mental math that is taught uses understanding of place value to model how mental arithmetic can be understood and done. These place value models build throughout the program to cover increasingly complex concepts.

Singapore is also known for its use of model drawing, often called “bar modeling” in the U.S. Model drawing is a systematic method of representing word problems and number relationships that is explicitly taught beginning in second grade and extending all the way to secondary. Students are taught to use rectangular “bars” to represent the relationship between known and unknown numerical quantities and to solve problems related to these quantities. This gives students the tools to develop mastery and tackle problems as they become increasingly more complex.

### Examples throughout the Math in Focus curriculum:



Kindergarten:

Student Book A, Part 2, page 42

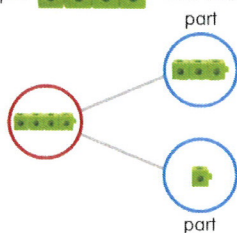
Pictorial models, combined with concrete hands-on activities, help Kindergarten students gain a meaningful understanding of number.

Learn

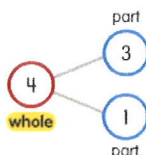
You can make number bonds with .

You can use a number train to make number bonds.

Sam put  into two parts.



How many are in each **part**?



3 and 1 make 4.

This picture shows a **number bond**.

Student Book A, page 30

Starting in 1st grade, students use number bonds to model part-part-whole relationships.

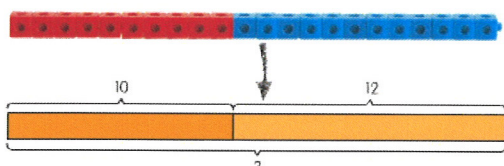
Learn

You can use bar models to help you add.

Mandy makes 10 granola bars.

Aida makes 12 granola bars.

How many granola bars do they make in all?



$$10 + 12 = 22$$

They make 22 granola bars in all.

**Check!**

$$22 - 10 = 12$$

$$22 - 12 = 10$$

The answer is correct!



2<sup>nd</sup> Grade:

Student Book A, page 96

Bar modeling is introduced in 2nd grade and carried throughout the Math in Focus curriculum.

Learn

Use **bar models** and addition or subtraction to solve 2-step real-world problems.

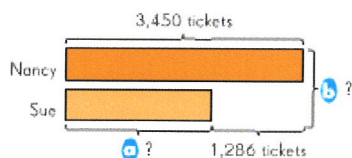
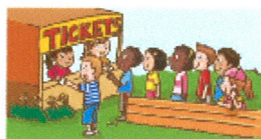
Nancy and Sue sold tickets for a concert.

Nancy sold 3,450 tickets.

Sue sold 1,286 fewer tickets than Nancy.

a How many tickets did Sue sell?

b How many tickets did they sell in all?



Sue sold fewer tickets than Nancy. So, use a comparison model.

a  $3,450 - 1,286 = 2,164$

Sue sold 2,164 concert tickets.

b  $3,450 + 2,164 = 5,614$

They sold 5,614 concert tickets in all.

**Check!**

$$2,164 + 1,286 = 3,450$$

$$5,614 - 2,164 = 3,450$$

The answers are correct.



3<sup>rd</sup> Grade:

Student Book A, page 122

As students tackle increasingly complex problems, they can use bar models to help them visualize, understand and solve.





## 5) Use appropriate tools strategically.

### From the Common Core State Standards:

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

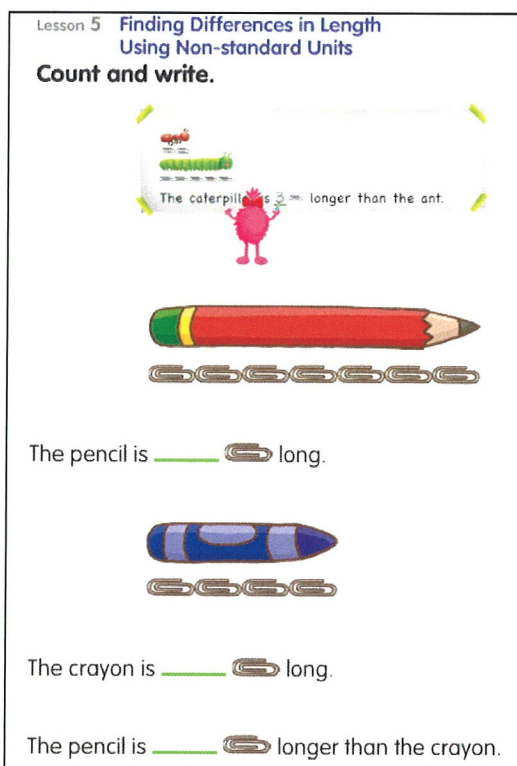
### How Math in Focus Aligns:

Math in Focus helps students explore the different mathematical tools that are available to them. New concepts are introduced using concrete objects, which help students break down concepts to develop mastery. They learn how to use these manipulatives to attain a better understanding of the problem and solve it appropriately. Math in Focus includes representative pictures and icons as well as thought bubbles that model the thought processes students should use the tools effectively. Additional tools are referenced and used in the program including clocks, money, dot paper, place value charts, geometric tools and figures.

### Examples throughout the Math in Focus curriculum:

Lesson 5 Finding Differences in Length  
Using Non-standard Units

Count and write.



The caterpillar is 3 units longer than the ant.

The pencil is 6 units long.

The crayon is 3 units long.

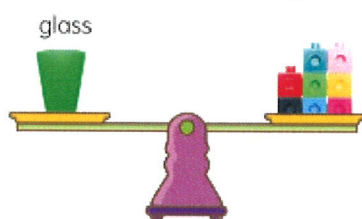
The pencil is 3 units longer than the crayon.


Kindergarten:

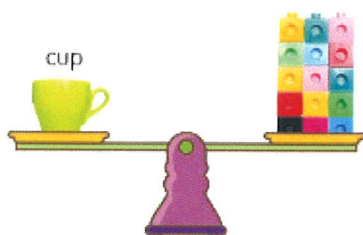



Learn

You can measure weight with objects.




The weight of the glass is about 8 .



The weight of the cup is about 15 .

The cup is heavier than the glass.

The glass is lighter than the cup.

The glass is as heavy as 8 .

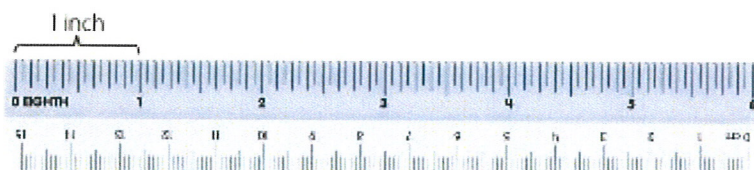


1<sup>st</sup> Grade:  
Student Book B, page 13

Learn

You can use inches to measure the length of shorter objects.

Inches are marked on this ruler. There are 12 inches in one foot.



What is inch?



It is a unit of length like the foot. You can use it to measure shorter objects.

The **inch** is a unit of length.  
**in.** stands for inch.  
Read 1 in. as one inch.  
Inch is used to measure shorter lengths.

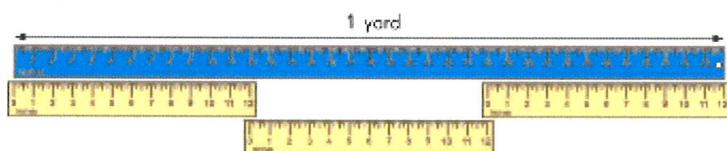


2<sup>nd</sup> Grade:  
Student Book B, page 111





# Use yards to measure length.



A yardstick is 3 times as long as a 12-inch ruler.

The **yard** is another standard customary unit of length. It is used for measuring long lengths and short distances. yd stands for yard.

1 yard (yd) = 3 feet (ft)

1 yard (yd) = 36 inches (in.)

A baseball bat is about 1 yard long.

A doorway is about 1 yard wide.

$$\begin{aligned} 1 \text{ ft} &= 12 \text{ in.} \\ 3 \text{ ft} &= 12 \times 3 \\ &= 36 \text{ in.} \end{aligned}$$

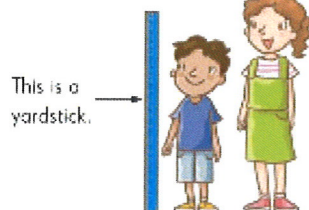


The height of a doorway is about 2 yards. The length of my garden is about 10 yards. The distance from my house to my neighbor's house is about 40 yards.



The boy is shorter than 1 yard.

The girl is taller than 1 yard.



The heights of both the boy and the girl are close to 1 yard. So, they are about 1 yard tall.



3<sup>rd</sup> Grade:

Student Book B, page 179

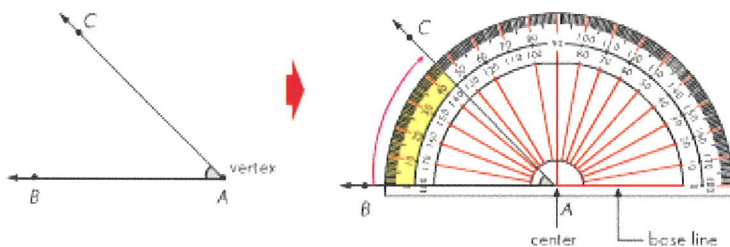
Learn

Use a **protractor** to measure an angle in **degrees**.



An angle measure is a fraction of a full turn. An angle is measured in degrees. For example, a right angle has a measure of 90 degrees. You can write this as  $90^\circ$ .

You can use a protractor to measure an angle.



**Step 1** Place the base line of the protractor on  $\overrightarrow{AB}$ .

**Step 2** Place the center of the base line of the protractor at the vertex of the angle.

**Step 3** Read the **outer scale**.  $\overrightarrow{AC}$  passes through the  $45^\circ$  mark. So, the measure of the angle is  $45^\circ$ .



Since  $\overrightarrow{AB}$  passes through the zero mark of the outer scale, read the measure on the outer scale.

4<sup>th</sup> Grade:  
Student Book A, page 88

Lesson

## 2.1 Using a Calculator

### Lesson Objective

- Use a calculator to add, subtract, multiply, and divide whole numbers.

Learn

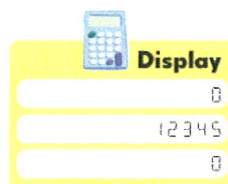
**Get to know your calculator.**

Turn on your calculator.

Follow the steps to enter numbers on your calculator.

To enter 12,345, press: **1 2 3 4 5**

To clear the display on your calculator, press: **C**



5<sup>th</sup> Grade:  
Student Book A, page 47



## 6) Attend to precision.

### From the Common Core State Standards:

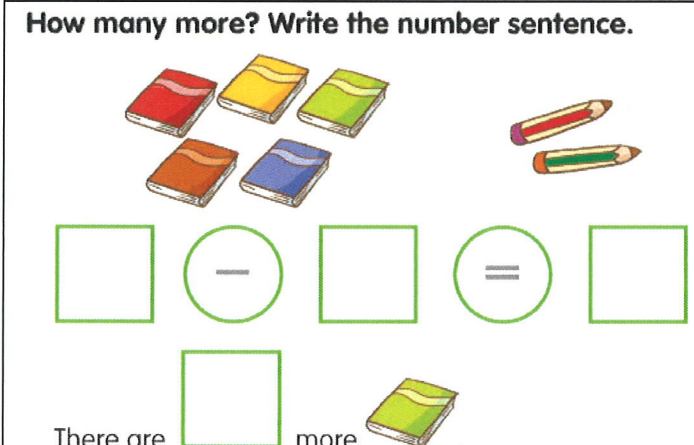
Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.


### How Math in Focus Aligns:

As seen in the Singapore Mathematics Framework, metacognition, or the ability to monitor one's own thinking, is key in Singapore's approach to mathematics. This is modeled for students throughout Math in Focus through the use of thought-bubbles, journal writing and prompts to explain your reasoning. When students are taught to monitor their own thinking, they are better able to attend to precision, as they consistently ask themselves, "does this make sense?" This questioning requires students to be able to understand and explain their reasoning to others as well as catch mistakes early on and identify when incorrect labels or units have been used.

### Examples throughout the Math in Focus curriculum:

**How many more? Write the number sentence.**



There are  more .

Kindergarten:

Student Book B, Part 2, page 28

Even in Kindergarten, when students are still learning to write, pictorial representations of units are used to emphasize their importance.

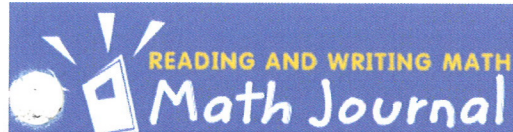


Why is it the least number?

1<sup>st</sup> Grade:

Student Book B, page 194

Thought bubbles throughout the Student Books prompt students to explain their answers.



$$758 - 35 = 732$$

Is the answer correct?

Explain why you think so.

Show how you would check it.

2<sup>nd</sup> Grade:

Student Book A, page 65

Getting the correct answer is not always the final goal.

Students are also asked to explain why or why not an answer is correct and how you would check to make sure.

This kind of thinking helps students establish the importance of precision and the need to understand how

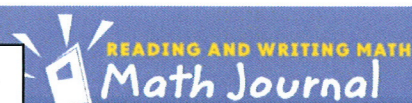
they solve a problem in order to evaluate whether or not it is correct.

3<sup>rd</sup> Grade Student Book B, page 211

3<sup>rd</sup> and 4<sup>th</sup> Grade:

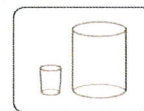
Math Journal activities ask students to consider how they would find an answer, requiring them to put their thought process into words. This reinforces the idea that the process that is used to get an answer is just as important as the answer itself, and that students must be able to explain how they got a solution precisely in order to ensure their result is reasonable.

4<sup>th</sup> Grade Student Book A, page 55



You are given an empty container and a cup.

**Explain how you would find the capacity of the container.**



### Example

These are the steps to find the factors of 12.

**1** Think of all the numbers that divide 12 exactly.

$$\begin{array}{ll} 12 \div 1 = 12 & 12 \div 4 = 3 \\ 12 \div 2 = 6 & 12 \div 6 = 2 \\ 12 \div 3 = 4 & 12 \div 12 = 1 \end{array}$$

**2** The factors are 1, 2, 3, 4, 6, and 12.

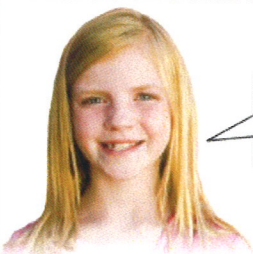
Write the steps for finding the common factors of 12 and 15.

Think of the multiplication tables.  
 $12 = 1 \times 12$   
 $12 = 2 \times 6$   
 $12 = 3 \times 4$





Find the difference between 1,005 pounds and 248 pounds.



Remember to write pounds in your answer.

Press	Display
<b>C</b>	0
<b>1 0 0 5</b>	1005
<b>- 2 4 8</b>	248
<b>=</b>	757

The difference between 1,005 pounds and 248 pounds is 757 pounds.

5<sup>th</sup> Grade:  
 Student Book A, page 48  
 Thought bubbles also provide reminders to students to consider units and labels as they solve problems.

## 7) Look for and make use of structure.

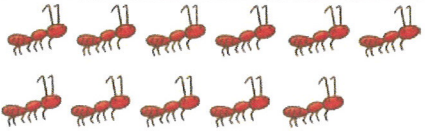
### From the Common Core State Standards:

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see  $7 \times 8$  equals the well remembered  $7 \times 5 + 7 \times 3$ , in preparation for learning about the distributive property. In the expression  $x^2 + 9x + 14$ , older students can see the 14 as  $2 \times 7$  and the 9 as  $2 + 7$ . They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see  $5 - 3(x - y)^2$  as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers  $x$  and  $y$ .

### How Math in Focus Aligns:

The inherent pedagogy of Singapore math allows students to look for and make use of structure. Place value is one of the underlying principles in Math in Focus. Concepts in the program start simple and grow in complexity throughout the chapter, year and from grade to grade. This helps students master the structure of a given skill, see its utility and advance to higher levels. Many of the models present in the program, particularly number bonds and bar models, allow students to easily see patterns within concepts and make inferences (Grade 1 number bond example shown here). For example, students are able to see particular properties of operations. As students move up into the grade levels, this level of structure becomes more advanced.

### Examples throughout the Math in Focus curriculum:



Count how many. \_\_\_\_\_

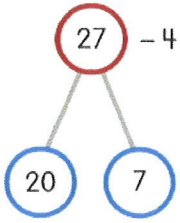
How many more to make 15? \_\_\_\_\_

44 Chapter 14

Kindergarten:


Student Book B, Part 1, page 44

Starting in Kindergarten, students learn that grouping objects can help solve problems.



$27 - 4$

$7 - 4 = 3$   
 $20 + 3 = 23$



So,  $27 - 4 = 23$ .

**Check!**

Remember,  $7 - 4 = 3$        $3 + 4 = 7$   
If  $27 - 4 = 23$ , then  $23 + 4$  should equal 27.  
The answer is correct.

2	3
+	4
27	

1<sup>st</sup> Grade:

Student Book B, page 102

Even first grade students learn to use number bonds to demonstrate the structure of numbers and understand properties.

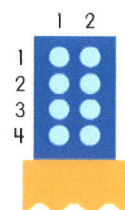


2<sup>nd</sup> Grade Student Book A, page 160

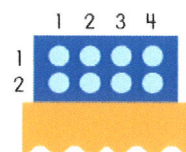
Learn

You can multiply numbers in any order.

$$4 \times 2 = 8$$



$$2 \times 4 = 8$$



$4 \times 2 = 2 \times 4$   
These are **related multiplication facts**.



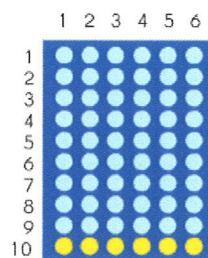
2<sup>nd</sup> and 3<sup>rd</sup> Grade:

Dot paper and thought bubbles help students recognize structure and how it can be used to break down and solve equations problems. These examples demonstrate how 2nd and 3rd graders learn that multiplication can be looked at in multiple ways.

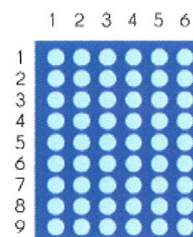
3<sup>rd</sup> Grade Student Book A, page 154

$$9 \times 6 = ?$$

Start with 10 groups of 6.



$$10 \times 6 = 60$$



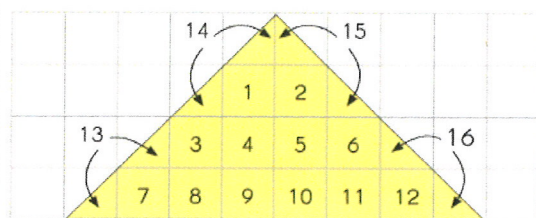
$$\begin{aligned} 9 \times 6 &= 10 \text{ groups of } 6 \\ &\quad - 1 \text{ group of } 6 \\ &= 60 - 6 \\ &= 54 \end{aligned}$$

$$\begin{aligned} 10 \times 6 &= 6 \times 10 \\ &= 60 \end{aligned}$$

$9 \times 6$  is the same as subtracting 1 group of 6 from  $10 \times 6$ .



Look at the triangle. Count the squares.



The area of the triangle is 16 square units.

4<sup>th</sup> Grade:

Student Book B, page 158

Students also learn to identify and use structure when dealing with geometry. In this 4th grade example, students must see how triangles relate to squares in order to determine the area of the object.

Look at the place-value chart.

	Hundreds	Tens	Ones
7			●●●●●●
$7 \times 10$		●●●●●●	
9			●●●●●●●●
$9 \times 10$		●●●●●●●●	
10		●	
$10 \times 10$	●		
12		●	●●
$12 \times 10$	●	●●	

What is the pattern when each number is multiplied by 10?



↓

	Hundreds	Tens	Ones
7			7
$7 \times 10$		7	0
9			9
$9 \times 10$		9	0
10		1	0
$10 \times 10$	1	0	0
12		1	2
$12 \times 10$	1	2	0

Each digit moves one place to the left when the number is multiplied by 10.



5<sup>th</sup> Grade:

Student Book A, page 52

In this 5<sup>th</sup> grade example, thought bubbles prompt students to see the pattern when each number is multiplied by 10. Place value charts help them visualize this pattern and solidify their learning.



## 8) Look for and express regularity in repeated reasoning.

### From the Common Core State Standards:

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation  $(y - 2)/(x - 1) = 3$ . Noticing the regularity in the way terms cancel when expanding  $(x - 1)(x + 1)$ ,  $(x - 1)(x^2 + x + 1)$ , and  $(x - 1)(x^3 + x^2 + x + 1)$  might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

### How Math in Focus Aligns:

A strong foundation in place value, combined with modeling tools such as bar modeling and number bonds, gives students the foundations they need to look for and express regularity in repeated reasoning.

In all the grades, operations are taught with place value materials so students understand how the standard algorithms work. Even the mental math that is taught uses understanding of place value to model how mental arithmetic can be understood and done. This allows students to learn shortcuts for solving problems and understand why they work.

Additionally, because students are given consistent tools for solving problems, they have the opportunity to see the similarities in how different problems are solved and understand efficient means for solving. Students continually evaluate the reasonableness of solutions throughout the program; the consistent models for solving, checking and self regulation, help them validate their answers.

### Examples throughout the Math in Focus curriculum:

How many more to make 10? Count and write.










Kindergarten:

Student Book A, Part 2, page 27

In activities like the one pictured here, Kindergarten students are asked to determine how many more train cars are needed to reach 10. By repeating this exercise with various quantities, students can see begin to see the connections between the numbers and develop shortcuts. For example, on the third line, once students determine that 7 more cars are needed to make 10, the teacher can point out that the train on the 4<sup>th</sup> car has one less than the 3<sup>rd</sup>, so they can add one to the number of cars that are needed to determine the answer.

**Learn** You can use place value to show numbers to 100.

Tens	Ones
9	8

90      8

$98 = 9 \text{ tens } 8 \text{ ones}$   
 $98 = 90 + 8$

**Guided Practice**  
 Use place value to find the missing numbers.

1

Tens	Ones
8	5

1<sup>st</sup> Grade:

Student Book B, page 184

Early on, students consistently think of numbers in terms of place value. This allows them to visually see the regularity of grouping by tens, hundreds, etc so they understand how operations work and can evaluate the reasonableness of their results.

**Learn** You can multiply when you have **equal groups**.

How many horses are there?

There are two ways to find the number of horses.

First, count the number of equal groups.  
 There are 3 groups.

Then, count the number of horses in each **group**.  
 There are 5 horses in each group.

Use **repeated addition** or **multiply** to find the number of horses.

$5 + 5 + 5 = 15$   
 $3 \times 5 = 15$

There are 15 horses in all.  
 $3 \times 5 = 15$  is a **multiplication sentence**.  
 You read it as **three times five is equal to fifteen**.

3 groups of 5 equal 15  
 3 fives = 15

$\times$  is read as **times**.  
 It means to **multiply**, or to put all the equal groups together.

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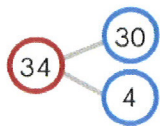
2<sup>nd</sup> Grade:

Student Book A, page 126

In 2nd grade, multiplication is introduced as repeated addition. This helps students understand how multiplication works so they can better apply their learning and check their work.

**Learn** Subtract 2-digit numbers mentally using the 'subtract the tens, then subtract the ones' strategy.

Find  $87 - 34$ .



$34 = 3 \text{ tens } 4 \text{ ones}$

**Step 1** Subtract 3 tens from 87.

$$87 - 30 = 57$$

**Step 2** Subtract 4 ones from the result.

$$57 - 4 = 53$$

So,  $87 - 34 = 53$ .

3<sup>rd</sup> Grade:

Student Book A, page 44

Mental math is an important part of the Math in Focus curriculum. Students develop a strong sense of place value, and understand the regularity of grouping by tens, hundreds, etc. This allows them to understand short cuts like the one taught in this 3rd grade lesson, where numbers can be broken apart by tens and ones to facilitate mental subtraction.

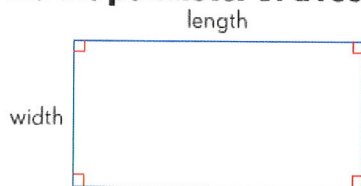


4<sup>th</sup> Grade:

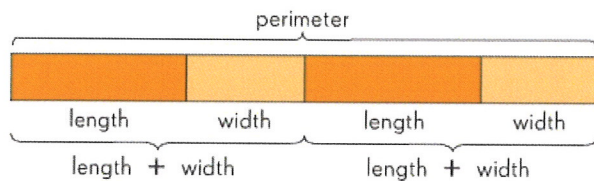
Student Book B,  
page 163

The consistent use of models throughout the program allows students to visually see connections that allow them to simplify problems and better understand how to solve. Here, a bar model demonstrates to 4<sup>th</sup> grade students that the length + the

**Learn** Find the perimeter of a rectangle using a formula.



Perimeter of rectangle = length + width + length + width  
= total length of all four sides



So, the length + width of a rectangle is equal to  $\frac{1}{2}$  of its perimeter.

You can use a model to show that the perimeter of the rectangle is the sum of its two lengths and two widths.



width of a rectangle is  $\frac{1}{2}$  the perimeter. Since they are able to see this relationship, they truly understand that relationship and can use it effectively.

5<sup>th</sup> Grade:

Student Book A, page 21

The consistent emphasis on place value allows students to see the structure of numbers and understand the reasoning behind number comparisons. In this 5<sup>th</sup> grade example, students look at the place value chart and see how with every number

**Learn** Compare numbers by using a place-value chart.

Which number is less, 237,981 or 500,600?

When comparing numbers, look at the value of each digit from left to right. Remember, '**>**' means '**greater than**' and '**<**' means '**less than**'.

Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Ones
2	3	7	9	8	1
5	0	0	6	0	0

Compare the values of the digits starting from the left. 2 hundred thousands is less than 5 hundred thousands. So, 237,981 is less than 500,600.

237,981 < 500,600



comparison, they can start with the digits on the left. The visual image of the place value chart helps solidify this process in the students' minds.

# **MATHEMATICAL CONTENT**



5A

# Math in Focus™

The Singapore Approach

7:9

## Correlation to the Common Core State Standards

*Math in Focus*™  
Grade 5

COMMON  
CORE

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Marshall Cavendish  
Education



HOUGHTON MIFFLIN HARCOURT

## Correlation of *Math In Focus*<sup>™</sup> to the Common Core State Standards

Attached are grade level correlations showing how closely *Math In Focus*<sup>™</sup> covers the skills and concepts outlined in the Common Core State Standards. But it is equally important to recognize the parallel assumptions behind the Common Core and *Math In Focus*<sup>™</sup>. In fact, the Singapore curriculum was one of the 15 national curriculums examined by the committee and had a particularly important impact on the writers because Singapore is the top performing country in the world and the material is in English.

Here are the parallel assumptions:

### 1. Curriculum must be focused and coherent:

#### **Common Core State Standards:**

*For over a decade, research studies of mathematics education in high performing countries have pointed to the conclusion that the mathematics curriculum in the United States must become substantially more focused and coherent in order to improve mathematics achievement in this country.*

*(Common Core State Standards for Mathematics, 3)*

*Math In Focus*<sup>™</sup> is organized to teach fewer topics in each grade but to teach them thoroughly. When a concept appears in a subsequent grade level, it is always at a higher level. For instance, first grade does not address fractions, second grade covers what a fraction is, third grade covers equivalent fractions and fractions of a set, fourth grade deals with mixed fractions, and addition of simple fractions, while fifth grade teaches addition, subtraction, and multiplication of fractions as well as division of fractions by whole numbers. This is the coherence and focus that the standards call for.



## **2. Teach to mastery**

### **Common Core State Standards:**

*In grade 2, instructional time should focus on four critical areas: (1) extending understanding of base-ten notation; (2) building fluency with addition and subtraction; (3) using standard units of measure; and (4) describing and analyzing shapes. (Common Core State Standards for Mathematics, 17)*

*In Grade 3, instructional time should focus on four critical areas: (1) developing understanding of multiplication and division and strategies for multiplication and division within 100; (2) developing understanding of fractions, especially unit fractions...;(3) developing understanding of the structure of rectangular arrays and of area; and (4) describing and analyzing two-dimensional shapes. (Common Core State Standards for Mathematics, 21)*

**Math In Focus™** has the identical structure. Rather than repeating topics, students master them in a grade level, and subsequent grades develop them to more advanced levels. Adding another digit is NOT an example. Moving from addition/subtraction in second grade to multiplication/division in third grade is such an example. Students continue to practice all the operations with whole numbers in every grade in the context of problem solving.

## **3. Focus on number, geometry and measurement in elementary grades**

### **Common Core State Standards:**

*Mathematics experiences in early childhood settings should concentrate on (1) number (which includes whole number, operations, and relations) and (2) geometry, spatial relations, and measurement, with more mathematics learning time devoted to number than to other topics. (Common Core State Standards for Mathematics, 3)*

**Math In Focus™** emphasizes number and operations in every grade K-5 just as recommended in the CCSS. The textbook is divided into two books roughly a semester each. Approximately 75% of Book A is devoted to number and operations and 60-70% of Book B to geometry and measurement where the number concepts are practiced. The key number topics are in the beginning of the school year so students have a whole year to master them.

#### **4. Organize content by big ideas such as place value**

##### **Common Core State Standards:**

*These Standards endeavor to follow such a design, not only by stressing conceptual understanding of key ideas, but also by continually returning to organizing principles such as place value or the properties of operations to structure those ideas.  
(Common Core State Standards for Mathematics, 4)*

**Math In Focus™** is organized around place value and the properties of operations. The first chapter of each grade level from second to fifth begins with place value. In first grade, students learn the teen numbers and math facts through place value. In all the grades, operations are taught with place value materials so students understand how the standard algorithms work. Even the mental math that is taught uses understanding of place value to model how mental arithmetic can be understood and done.

#### **5. Curriculum must include both conceptual understanding and procedural fluency.**

##### **Common Core State Standards:**

*The Standards for Mathematical Content are a balanced combination of procedure and understanding  
(Common Core State Standards for Mathematics, 8)*

**Math In Focus™** is built around the Singapore Ministry of Education's famous pentagon that emphasizes conceptual understanding, skill development, strategies for solving problems, attitudes towards math, and metacognition that enable students to become excellent problem solvers. The highly visual nature of the text and the consistent concrete to visual to abstract approach enables all students to both understand how procedures work and to fluently apply them to solve problems.



## 6. Mathematics is about reasoning

### **Common Core State Standards:**

*These Standards define what students should understand and be able to do in their study of mathematics....One hallmark of mathematical understanding is the ability to justify, in a way appropriate to the student's mathematical maturity.*  
(Common Core State Standards for Mathematics, 4)

**Math In Focus™** is famous for its model drawing to solve problems and to enable students to justify their solutions. In addition to journal questions and other explicit opportunities to explain their thinking, students are systematically taught to use visual diagrams to represent mathematical relationships in such a way as to accurately solve problems, but also to explain their thinking.

### Works Cited:

1. "Common Core State Standards For Mathematics" *Common Core State Standards Initiative | Home*. 2 June 2010. Web. 26 July 2010. <[http://www.corestandards.org/assets/CCSSI\\_Math%20Standards.pdf](http://www.corestandards.org/assets/CCSSI_Math%20Standards.pdf)>.

**Great Source Education**  
***Math in Focus, Grade 5* ©2009**

correlated to the

**Common Core State Standards for Mathematics**  
**Grade 5**

STANDARDS	DESCRIPTOR	PAGE CITATIONS
<b>STANDARDS FOR MATHEMATICAL PRACTICE</b>		
1.	Make sense of problems and persevere in solving them.	<p><i>This standard is covered throughout the program; the following are examples.</i></p> <p><b>SE/TE-A:</b> 3-4, 14, 15, 27-32, 34-35, 44, 46, 81, 94-95, 98-102, 103-108, 109, 110, 113, 122-123, 125, 129, 143, 151-153, 154-155, 159, 162, 169-174, 177-178, 179, 183-184, 185-187, 189-197, 199, 216, 230, 232-233, 237, 238-239, 241, 245, 262, 284-285, 286-289, 291, 293-295, 304-310, 311, 312, 315</p> <p><b>Workbook A:</b> 35B, 35C, 35D, 102A, 102B, 109A, 154C, 155A, 199A, 241A, 262A, 262B, 311B, 315G</p> <p><b>SE/TE-B:</b> 25, 35, 51-52, 66, 68-77, 81, 82-83, 115, 149, 153, 165-166, 179, 203, 223, 243-244, 251, 261, 273, 296</p> <p><b>Workbook B:</b> 25A, 74A, 81A, 95A, 115B, 153A, 179A, 223B, 297C</p>



STANDARDS	DESCRIPTOR	PAGE CITATIONS
2.	Reason abstractly and quantitatively.	<p><i>This standard is covered throughout the program; the following are examples.</i></p> <p><b>SE/TE-A:</b> 35, 65, 67-69, 98, 109, 112, 122, 127, 131-133, 158, 199, 230-235, 238-239, 243-245, 262</p> <p><b>Workbook A:</b> 69A, 69B, 136A, 235A, 262A</p> <p><b>SE/TE-B:</b> 59, 69-74, 82-83, 125, 133-135, 144-145, 151-152, 154-156, 163-168, 169-173, 175-178, 179, 180-181, 184, 191-194, 195-204, 223, 251</p> <p><b>Workbook B:</b> 74A, 168A, 168B, 173A, 178A, 178B, 179A, 194A, 204A, 204B, 223B, 228B, 228C, 228D, 297C</p>
3.	Construct viable arguments and critique the reasoning of others.	<p><b>SE/TE-A:</b> 14, 15, 81, 94, 95, 125, 129, 143, 168, 179, 216, 262, 291, 311</p> <p><b>SE/TE-B:</b> 66, 149, 203, 243, 244, 261, 273</p>
4.	Model with mathematics.	<p><i>This standard is covered throughout the program; the following are examples.</i></p> <p><b>SE/TE-A:</b> 47-50, 52, 55, 57, 60, 71, 73, 75, 78, 98-100, 102, 104, 108, 113, 133-135, 167, 188, 216, 224-225, 254, 257, 259, 281, 300</p> <p><b>Workbook A:</b> 50A, 63B, 102A, 102B, 108A, 262B</p> <p><b>SE/TE-B:</b> 36-39, 43-44, 46-47, 51, 53-55, 60-61, 63-64, 128, 136, 142, 167, 172, 175, 206, 212, 241, 260-261, 263-266, 282-283</p> <p><b>Workbook B:</b> 50A, 67A</p>

STANDARDS	DESCRIPTOR	PAGE CITATIONS
5.	Use appropriate tools strategically.	<p><i>This standard is covered throughout the program; the following are examples.</i></p> <p><b>SE/TE-A:</b> 47, 48-50, 55, 60, 73, 78, 98-100, 102, 104, 108, 113</p> <p><b>Workbook A:</b> 50A, 63B, 102A, 102B, 108A262B</p> <p><b>SE/TE-B:</b> 159, 162-164, 167, 170, 172, 175</p>
6.	Attend to precision.	<p><i>This standard is covered throughout the program; the following are examples.</i></p> <p><b>SE/TE-A:</b> 11, 14, 15, 47, 49, 50, 53, 55, 58, 60, 72, 73, 76, 78, 80, 81, 94-95, 123, 125, 128, 129, 133, 135, 143, 167, 168, 179, 188, 216, 224-225, 257-259, 262, 281, 291, 300, 311</p> <p><b>SE/TE-B:</b> 14, 44, 46, 49, 61, 64, 66, 79, 99, 105, 113, 128, 136, 142, 148, 149, 167, 172, 175, 196, 203, 206, 212-214, 220, 241, 243, 244, 249, 261, 273, 282</p>
7.	Look for and make use of structure.	<p><i>This standard is covered throughout the program; the following are examples.</i></p> <p><b>SE/TE-A:</b> 90-95, 99-102, 103-105, 109, 113, 153, 184, 191-194, 198, 222</p> <p><b>Workbook A:</b> 95A, 95B, 102A, 102B, 108A, 108B, 109A, 113D, 113E, 153A, 153C, 153D, 184A, 315E, 315F, 315H</p> <p><b>SE/TE-B:</b> 77-78</p> <p><b>Workbook B:</b> 80A, 80B</p>
8.	Look for and express regularity in repeated reasoning.	<p><b>SE/TE-A:</b> 35, 52-53, 57-58, 71-72, 75-76, 81, 109, 199, 262</p>



STANDARDS	DESCRIPTOR	PAGE CITATIONS
<b>STANDARDS FOR MATHEMATICAL CONTENT</b>		
<b>5.OA</b>	<b>OPERATIONS AND ALGEBRAIC THINKING</b>	
<b>Write and interpret numerical expressions.</b>		
1.	Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.	<b>SE/TE-A:</b> 55, 60-63, 64-66, 68, 73-74, 77-80, 92-95, 109, 113, 216-217, 227-228, 230, 235, 237-240, 242, 245 <b>Workbook A:</b> 63A, 81A, 81B, 95B, 108A, 108B, 109A, 113D, 225A, 240A, 241A, 315B, 315C, 315F, 315H
2.	Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them.	<b>SE/TE-A:</b> 90-93, 95, 99-101, 111, 113, 208-210 <b>Workbook A:</b> 95A, 95B, 218A, 315E
<b>Analyze patterns and relationships.</b>		
3.	Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane.	<b>SE/TE-A:</b> 51-54, 56-59, 70-72, 74-77, 81 <b>SE/TE-B:</b> 131- 135, 136, 138, 156 <b>Workbook B:</b> 138A, 153A

STANDARDS	DESCRIPTOR	PAGE CITATIONS
5.NBT	NUMBER AND OPERATIONS IN BASE TEN	
Understand the place value system.		
1.	Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.	<p><b>SE/TE-A:</b> 5-8, 9-10, 12-13, 16, 18, 20-21, 36, 38-39, 52-53, 57-58, 71-72, 75-76, 81</p> <p><b>Workbook A:</b> 15A, 15B, 19A, 24A</p> <p><b>SE/TE-B:</b> 7-14, 16-17, 18-19, 20-22, 23, 27-29, 36-39, 42, 43-44, 46-47, 51, 53-58, 60-61, 63-64</p> <p><b>Workbook B:</b> 17A, 22A, 42A, 50A, 59A, 59B, 67A</p>
2.	Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.	<p><b>SE/TE-A:</b> 51-54, 56-59, 70-72, 74-77, 81</p> <p><b>SE/TE-B:</b> 43-48, 50A, 60-65, 67A, 82-83, 118C</p> <p><b>Workbook B:</b> 50A, 67A, 118C</p>
3.	<i>Read, write, and compare decimals to thousandths.</i>	
3.a.	Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., $347.392 = 3 \times 100 + 4 \times 10 + 7 \times 1 + 3 \times (1/10) + 9 \times (1/100) + 2 \times (1/1000)$ .	<p><b>SE/TE-B:</b> 7-17, 17A, 23-25, 26-29</p> <p><b>Workbook B:</b> 17A, 25A, 25B, 118B</p>
3.b.	Compare two decimals to thousandths based on meanings of the digits in each place, using $>$ , $=$ , and $<$ symbols to record the results of comparisons.	<p><b>SE/TE-B:</b> 18-22, 27</p> <p><b>Workbook B:</b> 22A, 118B</p>
4.	Use place value understanding to round decimals to any place.	<p><b>SE/TE-B:</b> 20-22, 27, 29, 56-58, 68-74, 75-80, 82-83</p> <p><b>Workbook B:</b> 22A, 25B, 74A, 80A, 118B, 118C</p>



STANDARDS	DESCRIPTOR	PAGE CITATIONS
<b>Perform operations with multi-digit whole numbers and with decimals to hundredths.</b>		
5.	Fluently multiply multi-digit whole numbers using the standard algorithm.	<p><b>SE/TE-A:</b> 49, 51-63, 64-69, 91-95, 98-102, 103-104, 108, 109, 110, 112-113</p> <p><b>Workbook A:</b> 50A, 63A, 63B, 69A, 69B, 95A, 95B, 102A, 102B, 108A, 108B, 109A, 113C, 113D, 113E</p> <p><b>SE/TE-B:</b> 269-270, 272, 274, 287-295, 299, 301-302,</p> <p><b>Workbook B:</b> 247A, 297, 297A, 297B, 297C, 302B, 302C, 302D, 302E, 302G</p>
6.	Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models	<p><b>SE/TE-A:</b> 50, 70, 72-74, 77-81, 82-89, 92-94, 96-97, 100, 102, 104-105, 108, 111-113</p> <p><b>Workbook A:</b> 50A, 81A, 81B, 89A, 95A, 95B, 102A, 108B, 113C, 113E, 315E, 315G</p>
7.	Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.	<p><b>SE/TE-B:</b> 13, 17, 26, 36-42, 43-50, 51-59, 60-67, 68-74, 75-80, 81, 82-84</p> <p><b>Workbook B:</b> 17A, 25B, 42A, 50A, 59A, 59B, 67A, 74A, 80A, 80B, 81A, 118B, 118C, 302G</p>

STANDARDS	DESCRIPTOR	PAGE CITATIONS
5.NF	NUMBER AND OPERATIONS - Fractions	
Use equivalent fractions as a strategy to add and subtract fractions.		
1.	Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators.	<p><b>SE/TE-A:</b> 122-126, 127-130, 140-144, 145-149, 151-153, 154, 156, 158-159</p> <p><b>Workbook A:</b> 126A, 126B, 130A, 144A, 149A, 153A, 153B, 153C, 153D, 155A, 203B, 203C, 203D, 315E, 315F, 315G</p>
2.	Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers.	<p><b>SE/TE-A:</b> 122, 127, 129, 131-132, 134-135, 139, 140-141, 145-146, 150-153, 154-155, 159</p> <p><b>Workbook A:</b> 130A, 139A, 153A, 153B, 153C, 153D, 155A, 203D, 315F, 315G</p>



STANDARDS	DESCRIPTOR	PAGE CITATIONS
<b>Apply and extend previous understandings of multiplication and division to multiply and divide fractions.</b>		
3.	Interpret a fraction as division of the numerator by the denominator ( $a/b = a \div b$ ). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem.	SE/TE-A: 131-136, 138, 150, 157, 159 Workbook A: 136A, 139A, 153A, 153B, 203B
4.	<i>Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.</i>	
4.a.	Interpret the product $(a/b) \times q$ as $a$ parts of a partition of $q$ into $b$ equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$ .	SE/TE-A: 165-168, 169-174, 175-176, 177-180, 181-184, 191, 193-194, 196, 198, 200-203 Workbook A: 168A, 174A, 176A, 180A, 184A, 198A, 199A, 203C, 203D, 203E, 315E, 315F
4.b.	Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.	SE/TE-A: 165, 175, 203 Workbook A: 168A, 176A

STANDARDS	DESCRIPTOR	PAGE CITATIONS
<b>Perform operations with multi-digit whole numbers and with decimals to hundredths.</b>		
<b>5.</b>	<i>Interpret multiplication as scaling (resizing), by:</i>	
<b>5.a.</b>	Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.	SE/TE-A: 168, 280-281, 299, 301, 304, 313 Workbook A: 282, 301A
<b>5.b.</b>	Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying $a/b$ by 1.	SE/TE-A: 165-168, 175-176, 177-180 Workbook A: 168A, 176A SE/TE-B: 36-50 Workbook B: 42A, 50A
<b>6.</b>	Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.	SE/TE-A: 165, 167, 169-174, 177, 181-184, 192-197, 203 Workbook A: 174A, 184A, 199A, 203D, 203E
<b>7.</b>	<i>Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.</i>	
<b>7.a.</b>	Interpret division of a unit fraction by a non-zero whole number, and compute such quotients.	SE/TE-A: 185-189, 196-198, 201, 203 Workbook A: 189A, 199A, 203C, 203D, 203E, 315E
<b>7.b.</b>	Interpret division of a whole number by a unit fraction, and compute such quotients.	<i>This standard will be met in Grade 6.</i>
<b>7.c.</b>	Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem.	SE/TE-A: 185-189, 196-198, 203 Workbook A: 189A, 199A, 203D, 203E



STANDARDS	DESCRIPTOR	PAGE CITATIONS
5.MD	MEASUREMENT AND DATA	
<b>Convert like measurement units within a given measurement system.</b>		
1.	Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems.	<b>SE/TE-B:</b> 134, 290-296, 299, 301-302 <b>Workbook B:</b> 153A, 228B, 228D, 297A, 297B, 297C, 302C, 302D, 302E
<b>Represent and interpret data.</b>		
2.	Make a line plot to display a data set of measurements in fractions of a unit ( $\frac{1}{2}$ , $\frac{1}{4}$ , $\frac{1}{8}$ ). Use operations on fractions for this grade to solve problems involving information presented in line plots.	<i>Line Plots are covered to mastery in grades 3 and 4, opportunity to review can be found on page:</i> <b>SE/TE-B:</b> 153
<b>Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.</b>		
3.	<i>Recognize volume as an attribute of solid figures and understand concepts of volume measurement.</i>	
3.a.	A cube with side length 1 unit, called a “unit cube,” is said to have “one cubic unit” of volume, and can be used to measure volume.	<b>SE/TE-B:</b> 277-285, 287-290, 297, 299-301 <b>Workbook B:</b> 285A, 297A, 302C
3.b.	A solid figure which can be packed without gaps or overlaps using $n$ unit cubes is said to have a volume of $n$ cubic units.	<b>SE/TE-B:</b> 277-285, 287-288, 297, 299-300 <b>Workbook B:</b> 285A, 302C
4.	Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.	<b>SE/TE-B:</b> 277-285, 287-288, 297, 299-300 <b>Workbook B:</b> 285A, 302C

STANDARDS	DESCRIPTOR	PAGE CITATIONS
5.	<i>Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume.</i>	
5.a.	Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.	<b>SE/TE-B:</b> 277, 279, 280-283, 285, 287-288, 299 <b>Workbook B:</b> 285A, 302C
5.b.	Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems.	<b>SE/TE-B:</b> 286-297, 297A, 297B, 297C, 299, 301-302 <b>Workbook B:</b> 297A, 297B, 297C, 302C, 302D, 302G
5.c.	Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.	<b>SE/TE-B:</b> 278, 279, 281, 282, 284



STANDARDS	DESCRIPTOR	PAGE CITATIONS
5.G	GEOMETRY	
<b>Graph points on the coordinate plane to solve real-world and mathematical problems.</b>		
1.	Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., $x$ -axis and $x$ -coordinate, $y$ -axis and $y$ -coordinate).	<b>SE/TE-B:</b> 131-138, 156 <b>Workbook B:</b> 138A, 153A, 228B, 228D, 302E
2.	Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.	<b>SE/TE-B:</b> 131-138, 156 <b>Workbook B:</b> 138A, 153A, 228B, 228D, 302E
<b>Classify two-dimensional figures into categories based on their properties.</b>		
3.	Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category.	<b>SE/TE-A:</b> 257 <b>SE/TE-B:</b> 186-190, 195-199, 201-204, 211-214, 216-223, 224-228 <b>Workbook B:</b> 190A, 204A, 204B, 223A, 223B, 228C, 228D, 302F
4.	Classify two-dimensional figures in a hierarchy based on properties.	<b>SE/TE-A:</b> 257 <b>SE/TE-B:</b> 186-190, 195-199, 201-204, 211-214, 216-223, 224-228 <b>Workbook B:</b> 190A, 204A, 204B, 223A, 223B, 228C, 228D, 302F

# ACHIEVE WHITEPAPER



# Comparing the Common Core State Standards and Singapore's Mathematics Syllabus

## Introduction

Through the Common Core State Standards (CCSS) Initiative, states and territories have collaborated in the development of a common core of standards in English Language Arts and mathematics for grades kindergarten through twelve that are now being adopted by states. Designed not only for the purpose of providing strong, shared expectations, the Common Core State Standards will also allow adopting states to collectively create and share high-quality tools such as assessments, curricula, instructional materials (such as textbooks and software), and professional development programs.

As educators and policymakers review the CCSS in mathematics, they will want to consider the way these new standards compare to, and build on, existing standards in mathematics. This brief describes the comparison between the CCSS and Singapore's Mathematics Syllabus.

## Common Core State Standards in Mathematics

The K-5 standards provide students with a solid foundation in whole numbers, addition, subtraction, multiplication, division, fractions and decimals—which help young students build the foundation to apply more demanding math concepts and procedures successfully, and move into applications. They also provide detailed guidance to teachers on how to navigate their way through knotty topics such as fractions, negative numbers, and geometry, and do so by maintaining a continuous progression from grade to grade. Having built a strong foundation in K-5, students can move to more complex work in geometry, algebra and probability and statistics in the middle grades to gain a rich preparation for high school mathematics. Students who have completed 7<sup>th</sup> grade and mastered the content and skills through the 7<sup>th</sup> grade will be well prepared for algebra in grade 8. The high school standards call on students to practice applying mathematical ways of thinking to real world issues and challenges; they prepare students to think and reason mathematically across the major strands of mathematics, including number, algebra, geometry, probability and statistics. Note that the CCSS promote rigor not simply by including advanced mathematical content, but by requiring a deep understanding of the content at each grade level, and providing sufficient focus to make that possible.

The CCSS in mathematics lay out a vision for what all students need to master to be ready for credit-bearing college mathematics courses without remediation. Some of the high school standards are designated by a (+), indicating that they are above the college- and career-ready requirement but necessary for students to take advanced mathematics courses in high school such as calculus, advanced statistics, or discrete mathematics, and to be prepared for Science, Technology, Engineering, and Mathematics (STEM) coursework in college.

## Singapore's Mathematics Syllabus

Singapore's students have consistently been high performers, ranking first in the world in mathematics on the Trends in International Mathematics and Science Study (TIMSS) in 2003. As a result, researchers have explored Singapore's successful approach to mathematics instruction to identify features that could work in U.S. schools.<sup>1</sup> There are several components that account for Singapore's success, including "a highly logical national mathematics framework, mathematically rich problem-based textbooks, challenging mathematics assessments, and highly-qualified mathematics teachers."<sup>2</sup>

<sup>1</sup> *What the United States Can Learn From Singapore's World-Class Mathematics System (and What Singapore Can Learn from the United States): An Exploratory Study*. American Institutes for Research, 2005

<sup>2</sup> *Ibid*



Singapore's mathematics framework identifies five key aspects of mathematics learning as essential:

- ✓ Skills, such as estimation, mental calculation, arithmetic and algebraic manipulation, and handling of data;
- ✓ Numerical, geometrical, algebraic, and statistical concepts;
- ✓ Processes, or thinking skills;
- ✓ Metacognition, the monitoring of one's own thinking; and
- ✓ Attitudes regarding mathematics, including interest, confidence, and perseverance.

The Mathematics Syllabus, which is a part of Singapore's mathematics framework, describes the primary content expectations for each level through topic lists, which are similar to standards. Details are differentiated for students in different streams, or academic tracks. *All* students will study content through what is called the O-Level. Students who wish to pursue university coursework study A-Level mathematics. Because of its quality, the Singapore Syllabus was an important resource for the developers of the CCSS.

**More about Singapore's O- and A-Levels:** *The O-Level syllabi are titled Secondary 1, Secondary 2, and Secondary 3/4, which are equivalent to Grades 7, 8, and 9-10 in the U.S. Virtually all students in Singapore will take the O-Level exams, and 82% do well enough to qualify for junior college, which culminates in the A-Level exams. About 30% of students go on to junior college, while about 40% go on to polytechnic schools after completing the O-Level exams.*

## Achieve's Analysis

Achieve has analyzed the CCSS and the 2007 version of Singapore's Mathematics Syllabus for the primary and secondary levels to determine how they compare in terms of **rigor, coherence, and focus**. **Rigor** refers to the degree that sets of standards address key content that prepares students for success beyond high school. **Coherence** refers to whether the standards reflect a meaningful structure, revealing significant relationships among topics and suggest a logical progression of content and skills over the years. **Focus** refers to whether the standards suggest an appropriate balance in conceptual understanding, procedural skill, and problem solving with an emphasis on application and modeling; the standards should be teachable within a school year (or across four years of high school), and key ideas in a given grade or topic area should be clear. Standards that are rigorous, coherent, and focused provide better guidance to educators, students, and parents about desired learning outcomes than those that are not. Expert mathematics content analysts conducted a side-by-side comparison of the CCSS and the Singapore Mathematics Syllabus, looking particularly at the inclusion and treatment of mathematics topics at each grade level. This brief describes their findings.

## Major Findings

- ✓ The CCSS and Singapore Mathematics Syllabus describe similar levels of rigor. Where grade placement discrepancies occur between the two documents, they are usually within one year of each other.
- ✓ The CCSS and the Singapore Mathematics Syllabus are comparable in their coherence and focus, but there are a few key differences, which result in the CCSS providing greater detail and specificity.



## Detailed Findings

### Rigor

The CCSS and the Singapore Mathematics Syllabus describe expectations of comparable rigor at each grade level. The two documents contain similar expectations for what students should know and be able to do by the end of grades 4 and 8. In high school, though, the CCSS expect slightly more than the Singapore Mathematics Syllabus expects of all students.

- ✓ **Elementary grades:** Through the end of grade 4, the CCSS and the Singapore Mathematics Syllabus are comparable in rigor. For example, both documents require students completing grade 4 to be fluent at adding, subtracting, and multiplying with whole numbers; to understand and be able to apply place value; and to be able to classify simple two-dimensional geometric figures. These expectations form the basis for basic mathematical understanding in elementary school.
- ✓ **Middle grades:** There also are substantial similarities in the middle grades. Both documents include some of the major hallmarks of algebra—proportionality, linear expressions and inequalities, and using equations and inequalities to solve real-life and mathematical problems—which prepare students well for more advanced mathematics in high school. More specifically, the CCSS contain much of the same content in Grade 8 that is found in the Singapore Secondary 2 expectations.
- ✓ **High school:** The CCSS compare favorably with Singapore's secondary-level syllabus. The CCSS include nearly all of the topics found in the Singapore Mathematics Syllabus through the end of the O-Level, which are the requirements that all students must meet, including solving quadratic equations and the graphs of exponential functions, before advancing to further education and training. While the two documents describe similarly rigorous expectations, the CCSS cover some concepts not addressed in the Singapore Mathematics Syllabus for all students. When comparisons are made between the CCSS that are beyond the college- and career-ready minimum (designated with a (+) symbol), and the Singapore A-Level standards (which are beyond the minimum expectations of all students in Singapore), the Singapore standards are somewhat more rigorous, as they describe content typically found in a U.S. Calculus course, whereas the CCSS describe content up to that which is normally found in a U.S. Pre-Calculus course. As a result, advanced mathematical content is more comprehensive in the Singapore standards than in the CCSS.

### Coherence and Focus

Achieve's analysis indicates that the Common Core State Standards and the Singapore Mathematics Syllabus share some key traits of coherence and focus. For example, both documents place a heavy emphasis on number concepts and skills in the primary grades, in order to provide students with a solid foundation for work with more advanced content in later years. The expectations for each grade level are very similar in the two documents, and both describe coherent expectations that build gradually from the primary grades on. For example, both documents lay out a reasonable progression of addition, subtraction, multiplication, and division with fractions by connecting them to measures such as length, as well as by addressing them across grades four through seven. By grounding work with fractions in measurement, both sets of standards provide students with the opportunity to grasp an otherwise elusive topic through more concrete means. Furthermore, by situating the progression of content across four grades, the CCSS and Singapore Mathematics Syllabus emphasize depth of learning without unnecessarily repeating content from one year to the next.



While they are substantially similar in terms of focus, the development of key learning trajectories is more detailed in the CCSS than in the Singapore Mathematics Syllabus. The CCSS present content expectations with greater detail and specificity, more clearly describing the conceptual understanding students should have than the Singapore Mathematics Syllabus does. For example, the CCSS require students to understand place value and then to apply it in computation as in following Grade 1 standard, "Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using...strategies based on place value, properties of operations, and/or the relationship between addition and subtraction..." Here, Singapore provides little or no information about the connection between place value and addition and subtraction, stating only "include addition and subtraction within 100 involving a two-digit number and ones, a two-digit number and tens, two two-digit numbers," in its first grade (Primary 1) standard. As a result, teachers who use the CCSS will have greater guidance regarding both the skills and the concepts students should learn.

In short, the Common Core State Standards and the Singapore Mathematics Syllabus share many key traits of coherence and focus. Significant differences are found in the greater detail and specificity provided by the CCSS, which make them more useful for teachers.

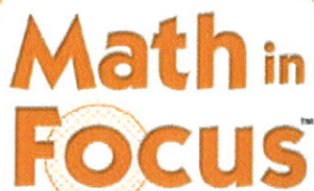
## Conclusion

Overall, the CCSS are well aligned to Singapore's Mathematics Syllabus. Policymakers can be assured that in adopting the CCSS, they will be setting learning expectations for students that are similar to those set by Singapore in terms of rigor, coherence and focus.

*Achieve is a bipartisan, nonprofit education reform organization that has worked with states, individually and through the 35-state American Diploma Project, for over a decade to ensure that state K-12 standards, graduation requirements, assessments and accountability systems are calibrated to graduate students from high school ready for college, careers and life. Achieve partnered with NGA and CCSSO on the Common Core State Standards Initiative and a number of its staff and consultants served on writing and review teams. Achieve thanks the Brookhill Foundation for its generous support in making this brief available, and providing educators and policymakers across the nation with a way to more deeply understand the CCSS through comparison to other well-known mathematics expectations. For more information about Achieve, visit [www.achieve.org](http://www.achieve.org)*



# **ADDITIONAL RESOURCES**



## **A study of the Singapore math program, *Math in Focus*- state test results**

*Report 404, December 2010*

### **Overview:**

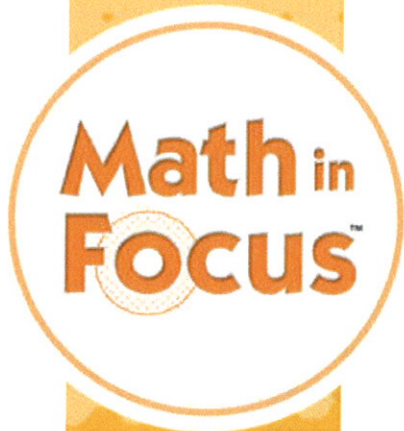
In a previous quasi-experimental, pretest/posttest study conducted by the Educational Research Institute of America in 2009-2010, fourth graders enrolled in Old Bridge Township School District, a large New Jersey school district using *Math in Focus* as part of a district pilot of the program, showed significant increases in math achievement over one academic year, as measured by the Stanford Achievement Test, Ninth Edition (SAT 9). The current study represents an extension of the 2009-2010 pilot study, as it utilizes the same research sample and timeline of program use. This analysis of 2010 state test scores sought to determine if similar gains would be seen if the outcome measure was the state mathematics test, the New Jersey Assessment of Skills and Knowledge (NJ ASK): Mathematics.

- 125 fourth graders in Old Bridge Township School District were engaged in the *Math in Focus* pilot during the 2009-2010 academic year.
- The remaining 553 students comprised the comparison group and used an alternative instructional mathematics program.
- In the year before the pilot of the Singapore math program *Math in Focus* began, all 678 students in all twelve elementary schools in the district used the same alternative program.

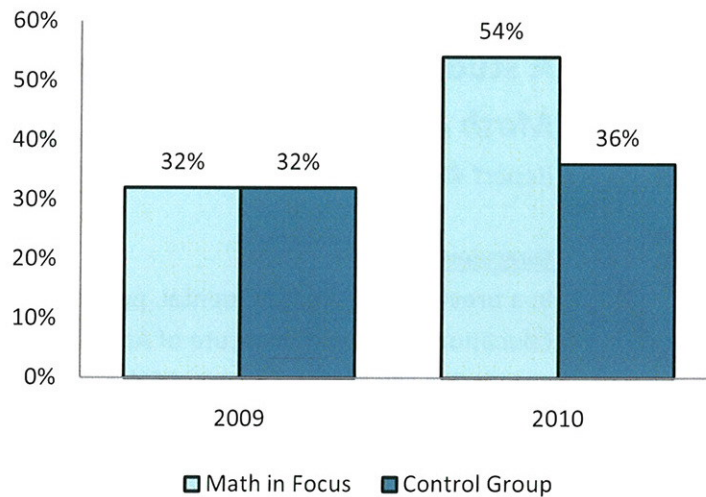
### **Grade 4 Findings:**

The results of the current study found that students using the *Math in Focus* program made significant gains on the state accountability test of mathematics achievement when compared to a control group of students from the same school district. In addition, significant gains were seen for students scoring at varying levels of mathematics proficiency prior to using the *Math in Focus* program.



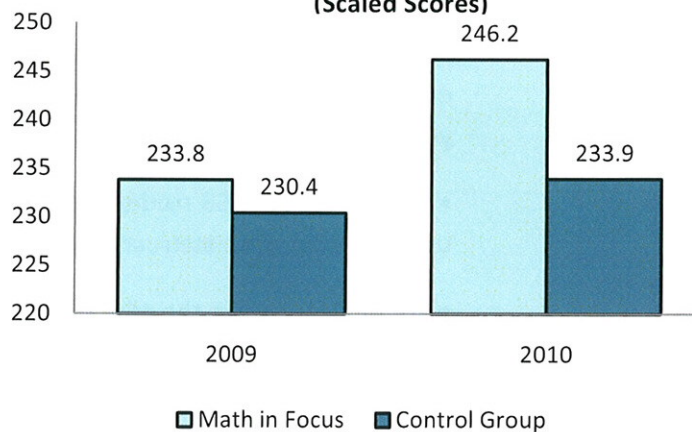


#### NJ ASK gains at the Advanced Proficient Level



The percentage of students scoring at the highest level, advanced proficient, increased by 20% for the *Math in Focus* group, while the control group increased by only 4%.

#### Student Performance on NJ ASK Over Two Years (Scaled Scores)



Overall, students using *Math in Focus* improved an average of 12.4 points, more than 3 times the average improvement that occurred across the remaining students in the district in the comparison group (average improvement = 3.5 points). The improvement in mathematics achievement among students using *Math in Focus* remained significant when researchers controlled for other predictors of academic performance, including teacher effects and student demographics. These results provide strong evidence of the instructional effectiveness of the *Math in Focus* program.

[www.greatsource.com/singaporemath](http://www.greatsource.com/singaporemath)



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# Math in Focus

The Singapore Approach

## Problem Solving In Singapore Math

written by Andy Clark

Ever since Singapore scored first in both 4th and 8th grades in the Trends in International Math and Science Study (TIMSS) comparison assessments in 1995, 1999, and 2003, and in the top three in 2007, math educators have been interested in the secret of Singapore's success. While many factors have been catalogued – a coherent national curriculum, teacher training, a public belief in the importance of math to the national economy – without fail, all descriptions emphasize the importance of problem solving in the Singapore curriculum. <sup>1</sup>(Leinwand 2008, AIR 2005, Lianghuo Fan & Yan Zhu 2007) This paper will consider how problem solving is taught in Singapore and why it has proven so successful.

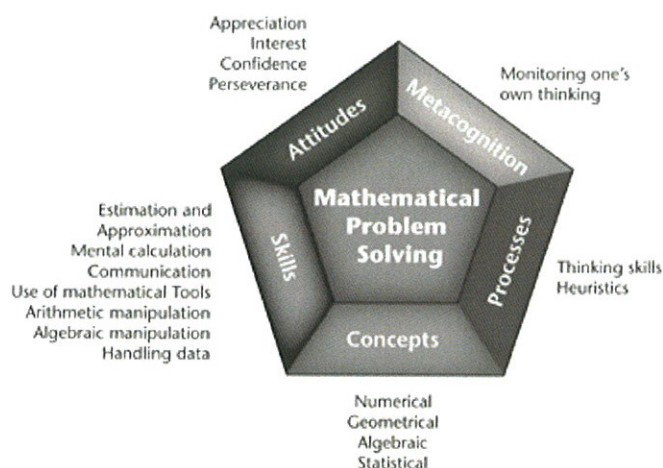
### A Greater Emphasis on Problem Solving

Interestingly, the first Singapore math curriculum, which was written in the 1980s, did not emphasize problem solving. It was not until 1991, and the writing of a new curriculum in 1992 that Singapore began emphasizing problem solving in its curriculum. As described on the Singapore Ministry of Education's Web site, problem solving is now the primary goal:

*Mathematical problem solving is central to mathematics learning. It involves the acquisition and application of mathematics concepts and skills in a wide range of situations, including non-routine, open-ended and real-world problems.*

*The development of mathematical problem solving ability is dependent on five interrelated components, namely, Concepts, Skills, Processes, Attitudes and Metacognition. (CPPD 2006)*

### Singapore's Mathematics Framework



The Singapore Ministry of Education uses a graphic to represent their vision for mathematics teaching: a pentagon, with problem solving in the center and these five interdependent, necessary elements surrounding it. Textbooks, written specifically to address this structure, provide constant support for understanding all five elements. Students are encouraged to consider how they think, how they communicate, and how they solve problems, so they can apply their skills to subsequent problems. In its latest efforts, the Ministry is working to increase student communication skills and metacognition during problem solving.



If this sounds familiar to educators in the United States, it should. While math education in the 1970's emphasized the importance of basic skills, in reaction to the "new math" of the fifties and sixties, the 1980's and 1990's saw the emergence of problem solving in national and state standards. In the 1980 *Agenda for Action*, and the 1989 and 2000 versions of *Principles and Standards*, the National Council of Teachers of Mathematics (NCTM) has similarly identified the centrality of problem solving to the math curriculum. (NCTM 1989, 2000) Most U.S. textbooks now include "problem solving strategies and problems." Additionally, American researchers have conducted many studies on how to develop problem-solving abilities. (Schoenfeld 1985, 1992; Krulik and Rednick 1987; Qin, Johnson, Johnson 1995)

### Differences in Performance Levels

Despite the increased emphasis on problem solving in the United States, students in Singapore continue to perform better in math. I would suggest there are five major reasons for this difference in performance:

1. Problem solving is embedded in Singapore texts, not as a separate activity but as central to every skill and concept discussion.
2. The problems that Singapore students work on are much more complex than those in typical American texts. Two- and three-step problems are the norm.
3. Non-routine, as well as routine problems are included in every grade level.
4. Students are taught specific problem-solving strategies in a carefully sequenced manner beginning in second grade. The most famous of these—model drawing—is used to solve word problems initially, but once acquired as a skill, it becomes useful for solving non-routine problems as well.
5. Student attitudes are addressed and supported.

Lets take a closer look at each of these critical differences.

### Embedded Problem Solving

Each time a new concept is introduced in the Singapore textbooks, problem solving is central to the discussion. Consider the concepts of perimeter and area. Just as they do in the U.S., third graders in Singapore learn about these concepts. But in Singapore, students immediately consider them from several perspectives. So rather than just calculating the perimeter of a square with side length 3, students are asked:

*A square of side 3 cm is formed using a piece of wire. The wire is straightened and then bent to form a triangle with equal sides. What is the length of each side of the triangle? (MIF 2nd grade)*

When learning about area of rectangles, students don't just calculate height times width, but are asked to find the side length of a square that has the same area as an 8 x 2 rectangle. In other words, whenever a skill or concept is taught, it is also applied in a problem setting. The problem may not even look like other problems that are modeled. Students learn that a single skill can be applied to a wide range of problems.



## Complex Word Problems

Complex word problems are the norm in the Singapore textbooks. Yet students begin with very simple problems.

*There are 5 boxes of pencils. Each box has 12 pencils. How many pencils altogether?*

By the end of the third grade chapter, however, they solve problems such as this one:

*Shawn and Trish scored 36 goals in all. Shawn scored three times as many goals as Trish. How many goals did Trish score?*

or

*Flo saves 4 times as much money as Larry. Maria saves \$12 less than Flo. Larry saves \$32. How much does Maria save?*

In other words, the Singapore textbooks take seriously the proposition that the purpose of math is to solve complex problems. Multiple-step word problems are introduced in the primary grades and become increasingly challenging in the higher grades. Thoughtfulness is displayed, though, in the simplicity of the initial problems. As students develop confidence, they tackle more complicated problems, including non-routine ones.

## Solving Non-routine Problems and the Use of Heuristics

In addition to complex word problems, Singapore curriculum emphasizes non-routine problems—those that go beyond the application of specific computation. Solutions to such problems often require a number of different strategies, or heuristics. Before looking at specific problems from the curriculum, let's review the history of non-routine problems.

The NCTM (2000) defines problem solving as “engaging in a task for which the solution method is not known in advance” (p. 52). Johnson, Herr, Kysh (2004) define problem solving as “knowing what to do when you don't know what to do” (p. 3). But perhaps the best definition may come from the math teacher who did the most in the last century to include problem solving in the math curriculum, George Polya. Polya (1965) says, “solving a problem means finding a way out of a difficulty, a way around an obstacle, attaining an aim which is not immediately attainable.”

Polya went on to describe problems, now called non-routine problems, which did not lend themselves to mechanical application of one or more straightforward algorithms. These are problems in which you have to figure out *how* to proceed, as well as to calculate successfully.

The Singapore textbook writers are very familiar with Polya's work and include it in manuals designed for teachers. (Huat & Huat 2006) In particular they cite the 4-step model that Polya recommended: understand the problem, devise a plan, carry out the plan, and reflect on the solution. While this same structure is in many American textbooks as well, it is the special attention that Singapore math pays to steps 1 and 4 that may set it apart. In the first step, students connect a problem to prior problems. In the reflection stage students consider *Would alternate solutions take a shorter time? Can the method be applied to other problems?* (Huat 2006)



The Singapore curriculum and textbooks build on the work of Polya by teaching specific strategies for problem solving, what Polya called heuristics. Many of these—such as “look for a pattern,” “draw a picture,” “simplify the problem”, and “work backwards”, are included in American texts. But too often, American texts teach these strategies by “type.” That is, students learn “look for a pattern” and then they are presented with problems that can be solved in that way. In Singapore, students are instead encouraged to consider which strategy will work best for a particular problem. They are introduced to the strategies or heuristics and then given a variety of non-routine problems to solve.

Let’s look now at a third grade example.

*Mr. King has a total of 19 geese, chickens and ducks on his farm. He has 3 more chickens than geese. He has 2 fewer ducks than geese. How many ducks does he have?*

Try to solve this. Then imagine a third grade student solving it. Singapore students are taught to draw models to represent the situation. Students might use the most basic strategy of guess and check or they may try it more systematically. For instance, one student took 19 counters, assigning 1 to the duck 3 to the geese and 6 to the chickens. The student then added a counter to each row until all 19 had been used.

Other students used a “work backwards” strategy. They know that if they take away 5 chickens and 2 geese, then they will be left with three equal piles. Since there are 12 counters or animals left, this means there must be 4 of each, hence 4 ducks. Notice that many of these strategies lead to the generalization students will use later in algebra to solve the problem. Encouraging multiple approaches and evaluating their effectiveness is the essence of good problem solving.

### **Model Drawing**

The most famous and developed of the heuristics (strategies) taught in Singapore is “model drawing,” sometimes called “bar modeling” in the U.S. Beginning in 2nd grade, students are taught to use rectangular shapes to model a word problem. These models:

- help students visualize abstract math relationships through pictorial representations.
- use rectangular blocks because they are easily divided.
- can be used before students know algebraic solutions, and can be used to model algebraic relationships.

Dr. Yeap Ban Har of Singapore’s National Institute of Education describes the model method this way:

*The use of “model method” provides students with a means to handle information, deal with complexity, and at the same time, communicate their thinking through the use of visual models which they can manipulate.*



These models begin quite simply and typically model word problems. In the Grade 2 problem at right, a rectangle, divided into two parts, models a simple subtraction situation. There are 20 eggs, 7 are duck eggs. How many are chicken eggs? Other models include simple multiplication and division. By 5th grade, however these models increase significantly in complexity. In fact, model drawing proves to be a powerful tool for non-routine as well as routine problems. Consider the following problem:

*25% of the fish in a bowl are guppies.  
The same number of guppies as were  
originally in the bowl are then added.  
What percentage of the bowl is now guppies?*

Initially students are confused because there do not appear to be enough numbers. Yet a simple diagram can help. Students draw a rectangle, color one-fourth as guppies. Then they add a length to the rectangle that is the same size as the guppies portion. The picture shows clearly that 2 of the 5 parts of the bowl are guppies or 40%.

Careful attention to the teaching of heuristics, to moving from simple to complex problems, and to sequencing the problems in such a way as to move from routine to non-routine problems is a hallmark of the textbooks. Still, problem-solving does not account for all the differences in performance between students in Singapore and the US. Attitudes toward mathematics and the development of metacognitive skills also play a role.

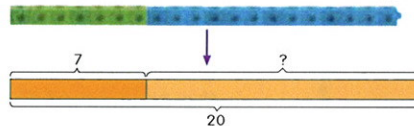
## Building Enthusiasm and Metacognition

In Singapore, efforts to develop positive attitudes and to improve metacognitive skills are evident in all aspects of mathematics learning, including classroom learning materials and the information sent home to students and families. Teachers are encouraged to solve mathematics problems themselves, so they develop a deeper understanding. Developing positive attitudes towards problem solving is a central focus—one that helps all students to feel better about math.

Ironically, at the same time as many states in the U.S. are depending exclusively on multiple choice tests, the Singapore assessment system is moving to incorporate more open ended assessment. Only 20% of the 6th grade placement test includes multiple-choice items. But Singapore's educators believe that constructed response problems "require students to be able to read reasonably complex paragraphs. In addition students are expected to be able to communicate their thinking and methods by showing how they reason and arrive at the answer." (Yeap 2008)

### Finding 1 set of objects from a group

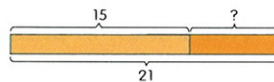
- 3 Ahmad bought 20 chicken eggs and duck eggs. There were 7 duck eggs. How many chicken eggs were there?



$$20 - 7 = 13$$

There were 13 chicken eggs.

- 4 A school set up a new aquarium. There are 21 fishes in it. 15 fishes were given by the parents. The rest were bought by the school. How many fishes did the school buy?



$$21 - 15 = 6$$

The school bought 6 fishes.

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My Pals Are Here: Maths,  
Grade 2, page 62



When addressing the winners of the high school math prizes, Ms Grace Fu, Senior Minister of State, Ministry of National Development and Ministry of Education Learning of Mathematics told the winners:

*Mathematics will imbue you with problem-solving skills such as the ability to understand a problem, identify the relevant information, look for relationships and patterns, make your own conjectures and apply mathematical knowledge and tools to solve or prove them. Mathematics, thus, provides good opportunities for the training of the mind to think critically and adapt to new situations, something that will be valuable to your future careers. (2008)*

In Singapore, a country lacking natural resources and half the size of New York City, human capital is the most precious resource. The Singapore curriculum and textbooks recognize that developing problem-solving skills and creativity is a requirement for the 21st century.

This belief, in addition to careful attention to teaching and cultivating problem-solving skills, are lessons worth considering as we try to make our students competitive in the global marketplace in which they live.



**Andy Clark** is a former elementary and junior high school teacher. He recently retired as the K-12 Math Coordinator for Portland Public Schools, an urban district that outperformed the state of Oregon and closed the achievement gap. Andy is coauthor of a number of math programs, including *Every Day Counts®: Calendar Math Pre-K—6*, *Algebra Readiness 6 & Up*, *Partner Games K-6*, and *Math in Focus™: The Singapore Approach K-5*.



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# Math in Focus

Singapore Math

by Marshall Cavendish

## Singapore Math:

## Place Value in *Math in Focus*<sup>TM</sup>

There are several skills that are critical to mathematics success in the elementary grades. Among them are an understanding of number, number sense, and place value. Place value is the system of writing numerals in which the value of the digit is determined by its position, or relationship to the other digits. These values are multiples of a common base of 10 in our decimal system. In Singapore, where students consistently outperform American students in math, place value is considered a foundational skill for all mathematics learning.

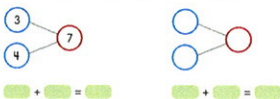
The U.S. edition of Singapore's math curriculum, *Math in Focus*, stresses mastery, coherence, and focus. Students master foundational math skills early and carry those skills with them as they progress through the grades. With each new concept learned, students build on what they have already learned. This systematic approach to mathematics is particularly evident in the program's approach to teaching place value. From grade level to grade level, place value is developed, honed, and applied to operations. As a result, numbers are demystified and students excel in mathematics because, for them, numbers become tools for solving problems rather than obstacles that cause frustration.

Throughout *Math in Focus*, a concrete-pictorial-abstract approach encourages students to use place value blocks, chips, charts, and strips to create an association between the physical representation of numbers, the number symbol, and the number name, and later to perform arithmetic operations using these materials. These concrete and pictorial representations add meaning to numbers so that when students progress to the abstract, they have an understanding of what the numbers stand for. This way students experience problem solving as more than just a process, there is real meaning behind the elements of a problem.

In Grade 1, place value is developed in a coherent, focused manner to stress the importance of the number 10 and help students master it. Grade 1, Chapter 1 begins with the study of the set of numbers from 0 to 10. This is followed by an introduction to number bonds, showing the additive relationship between numbers less than 10 and, in later chapters, adding and subtracting up to 10. In these chapters, students come to understand 10 as an anchor number for future applications of sums and differences with 10 and multiples of 10.

Add.  
Use number bonds to help you.

7 How many monkeys are there in all?



There are 7 monkeys in all.

You can add in any order.

$$4 + 3 = 7$$

4 added on to 3 is equal to 7  
3 added on to 4 is also equal to 7.



50 Chapter 3 Addition Facts to 10



Place value is introduced with numbers to 20, in a chapter that examines all “teen” numbers (11 to 19) as “10 plus numbers.” This helps students master two major concepts that will play a pivotal role in their future mathematics learning:

1. Seeing all two-digit numbers as composed of tens and more
2. Understanding expanded notation— $10 + 1$  or 1 ten plus 1,  $10 + 2$  or one ten plus two,  $10 + 3$  or one ten plus three, and so on.

Next comes a chapter focused on practice with addition and subtraction to 20, which helps to solidify students’ understanding of 10 as an anchor number, the place value of numbers to 20, and adding and subtracting of numbers to 20.

**Guided Practice**

Make a 10.  
Then add.  
Use number bonds to help you.

1  $9 + 5 =$



2  $8 + 7 =$



**Let's Practice**

Make a 10.  
Then add.

1  $9 + 4 =$



2  $7 + 9 =$



3  $9 + 8 =$



4  $8 + 3 = 10 +$    
 $=$

5  $6 + 8 =$   +   
 $=$

**ON YOUR OWN**  
Go to Workbook A:  
Practice 1, pages 191–196

Lesson 1 Ways to Add 197


Grade 1A, Chapter 8, page 197

By the second half of Grade 1, students using **Math in Focus** have developed a solid understanding of place value with numbers to 20. As the year progresses, the concept of place value continues to be developed with a chapter devoted to numbers to 40, followed by addition and subtraction to 40. In this chapter students begin their study of regrouping, which is clearly based on their earlier study of place value up to 40. Previously, students learned to make a ten and learned how to use number bonds. These foundational skills enable them to begin working with multi-digit numbers. Focusing only on numbers to 40 allows students to associate numerals with easily managed quantities of physical materials while the place value concept is developed.




In Grade 1, the study of place value concludes with the study of numbers to 100 with addition and subtraction. At this point, students are fully prepared to expand their understanding to three digits because of the gradual, strategic way in which it has been introduced, understood, and practiced. In addition, first-grade students are taught mental math in a chapter which calls on their knowledge of number bonds with 10 and multiples of 10 to solve mental computation problems.

**You can add tens mentally using the 'add the tens' strategy.**  
Find  $23 + 10$ .  
Group 23 into tens and ones.



**Step 1** Add the tens.  $20 + 10 = 30$   
**Step 2** Add the result to the ones.  $3 + 30 = 33$   
So,  $23 + 10 = 33$ .

**Guided Practice**  
**Add mentally.**  
4 Find  $15 + 20$ .  
Group 15 into tens and ones.



**Step 1** Add the tens.  $10 + \underline{\quad} = \underline{\quad}$   
**Step 2** Add the result to the ones.  $\underline{\quad} + \underline{\quad} = \underline{\quad}$   
So,  $15 + 20 = \underline{\quad}$


5 Find  $29 + 10$ .  $\underline{\quad}$

138 Chapter 14 Mental Math Strategies

Grade 1B, Chapter 14, page 138


In Grade 2, students study how to count, read and write numbers up to 1,000, and learn to add and subtract numbers to 1,000 with and without regrouping. Here place value is key to understanding why the algorithms work. Students learn how the place value ideas they learned in Grade 1 can be applied to the thousands place by using place value blocks. In addition, there is a chapter that teaches mental math strategies for adding and subtracting three-digit numbers without the use of paper and pencil. This solidifies students' understanding of place value.

**You can add tens to a 3-digit number mentally using the 'add the tens' strategy.**  
Find  $213 + 50$ .  
Group 213 into tens, and hundreds and ones.



**Step 1** Add the tens.  $10 + 50 = 60$   
**Step 2** Add the result to the hundreds and ones.  $203 + 60 = 263$   
So,  $213 + 50 = 263$ .

**Guided Practice**  
**Add mentally.**  
Use number bonds to help you.  
10 Find  $351 + 40$ .  
Group 351 into tens, and hundreds and ones.



**Step 1** Add the tens.  $\underline{\quad} + 40 = \underline{\quad}$   
**Step 2** Add the result to the hundreds and ones.  $301 + \underline{\quad} = \underline{\quad}$   
So,  $351 + 40 = \underline{\quad}$

11  $237 + 50 = \underline{\quad}$  12  $613 + 70 = \underline{\quad}$

12 Chapter 10 Mental Math and Estimation

Grade 2B, Chapter 10, page 12

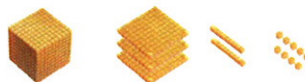


In Grade 3, the place value chart is extended to the ten thousands place. Predictably, the study of addition and subtraction in the ten thousands place follows. The depth of development of place value in Grade 3 directly correlates to the depth of development in Singapore Math. The chapter teaches the following skills:

1. Count, read, and write numbers to 10,000.
2. Learn to count by ones, tens, hundreds, and thousands.
3. Show representations of numbers up to 10,000 using place value charts and place value strips.
4. Identify the place value of each digit in the number and express the number in standard, word, and expanded forms.
5. Compare and verbally describe sets of numbers using the terms least and greatest.
6. Write five-digit numbers in increasing or decreasing order.
7. Apply the number and place value concepts to identify and complete number patterns and find missing numbers on a number line.

### Guided Practice

How many are there? Find the missing numbers.



Thousands	Hundreds	Tens	Ones

1  $1,329 =$   thousand  hundreds  tens  ones

2  $1,329 =$   +  +  +

3 In 1,329,  
the digit  is in the thousands place.  
it stands for .  
its value is .

**Find each missing number or word.**  
**Use base-ten blocks to help you.**

4 In 2,548,  
the digit  is in the hundreds place.  
the digit 4 stands for .  
the value of the digit 8 is .

5 In 2,562, the values of the digit 2 are:

$\xleftarrow{\quad}$  2, 5 6 2  $\xrightarrow{\quad}$



Throughout **Math in Focus**, students develop fluency in understanding numbers in multiple place value representations; for example, two thousand five hundred is 25 hundreds or 250 tens.

In the later chapters of Grade 3, students explore strategies for mental math and addition and subtraction for numbers to 10,000. These are the final chapters exploring place value addition and subtraction. In Grades 4 and 5, students continue to explore place value, but now with an emphasis on multiplication and division, and decimals.

Hundreds	Tens	Ones
$200 \times 3 = 600$	$30 \times 3 = 90$	$2 \times 3 = 6$

**Step 3**  
Multiply the hundreds by 3.  
2 hundreds  $\times$  3  
= 6 hundreds.

$2 \times 3 = 6$
$30 \times 3 = 90$
$200 \times 3 = 600$
<b>Total = 696</b>

**Multiplying with regrouping in hundreds, tens, and ones**  
Find  $125 \times 7$ .

**Step 1**  
Multiply the ones by 7.  
 $5 \text{ ones} \times 7 = 35 \text{ ones}$   
Regroup the ones. 35 ones = 3 tens 5 ones

**Step 2**  
Multiply the tens by 7.  
 $2 \text{ tens} \times 7 = 14 \text{ tens}$   
Add the tens. 14 tens + 3 tens = 17 tens  
Regroup the tens. 17 tens = 1 hundred 7 tens

**Step 3**  
Multiply the hundreds by 7.  
 $1 \text{ hundred} \times 7 = 7 \text{ hundreds}$   
Add the hundreds. 7 hundreds + 1 hundred = 8 hundreds  
So,  $125 \times 7 = 875$ .

$125$
$\times 7$
<b>875</b>

72 Chapter 3 Whole Number Multiplication and Division

Grade 4A, Chapter 3, page 72

In Grade 4, students extend their understanding of the place value system to the hundred thousands place and decimals to hundredths. Again, fluency with place value is emphasized as students learn to understand. For example, students see 38.2 as 38 and 2 tenths, or 3 tens and 82 tenths, or 382 tenths, and can then apply their knowledge of place value to multiplication, division, and mental math.

**Guided Practice**

Express each of these as a decimal.

1  $\frac{4}{100}$  oz =  oz      2  $\frac{6}{100}$  in =  in

3 five hundredths =       4 8 hundredths =

Find the decimals that the shaded parts represent.

5       6

Find the decimal for each point on the number line.

7

Find equivalent tenths and hundredths.

$\leftrightarrow$   $\leftrightarrow$

$\frac{10}{100}$  is equal to  $\frac{1}{10}$  or 0.1  
10 hundredths = 1 tenth  
You can regroup 10 hundredths as 1 tenth.

14 Chapter 7 Decimals

Grade 4B, Chapter 7, page 14





## Conclusion

In ***Math in Focus*** place value is presented in a systematic and focused manner. From the very beginning students understand numbers as a system of tens. This system lays the foundation for all future lessons including computation skills, problem solving, the decimal system, and more.

Mastery of math in the elementary grades is key for student success in the classroom and in the 21st century working world they will eventually enter. Since the 1980s, students in Singapore have consistently outscored U.S. students in math. Now, with ***Math in Focus***, U.S. students can learn using the same effective manner with which students in Singapore have had such success, preparing them for a lifetime of deep mathematics understanding and real-world problem solving.



## About the Author

**Patsy F. Kanter** is an author, teacher, and international math consultant. She worked as the Lower School Math Coordinator and Assistant Principal at Isidore Newman School in New Orleans, Louisiana, for 13 years. Kanter is the author of a number of mathematics programs, including ***Afterschool Achievers: Math Club (K–8)*** and ***Summer Success®: Math (K–8)***, and co-author of ***Every Day Counts®: Calendar Math, Partner Games (K–6)***, and ***Practice Counts (1–6)***. She is a consulting author for ***Math in Focus***, which is the U.S. version of Singapore's *My Pals Are Here! Maths*. Both programs are published by Marshall Cavendish, Singapore (part of Times Publishing Limited).



# Math in Focus™

## Singapore Math

by Marshall Cavendish

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